AN ROINN OIDEACHAIS

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1950.

MATHEMATICS—Geometry—Honours.

WEDNESDAY, 7th JUNE.—Morning, 10 to 12.30.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. If the polar of a point P, with respect to a given circle, passes through R, prove that the polar of R passes through P.

P and Q are inverse points with respect to a circle of centre O. R is any point on the line through Q perpendicular to OQ and H is the orthocentre of the triangle POR. Prove that the triangle PHR is such that each side is the polar of the opposite vertex.

[40 marks.]

2. (*a*) If two circles cut one another orthogonally, prove that any diameter of one circle is cut harmonically by the other circle.

(*b*) If a circle cuts each of two circles of a system of non-intersecting coaxal circles orthogonally, prove that it cuts all circles of the system orthogonally and that it passes through the limiting points of the system.

[40 marks.]

Or,

2. A transversal cuts the sides BC, CA, AB of a triangle ABC at L, M, N respectively, prove that

$$\frac{BL}{LC}\cdot\frac{CM}{MA}\cdot\frac{AN}{NB}=-1.$$

If the tangents to the circumcircle of the triangle ABC at the vertices A, B, C meet the sides produced in D, E, F respectively, prove that D, E, F are collinear.

[40 marks.]

3. The co-ordinates of the vertices of a triangle are (0, 0), (3, —1), (2, 4). Find the co-ordinates of (i) the orthocentre, (ii) the centre of the circumscribing circle.

[42 marks.]

4. Find (i) the equation of the circle which passes through the points $(-1, 4)$, $(1, 2)$ and has its centre on the straight line $3x-y-3=0$, (ii) the equations of the circles which pass through the points $(-1, 4)$, $(1, 2)$ and which touch the straight line $3x-y-3=0$.

[42 marks.]

5. Show that the straight line $y=mx+\frac{a}{m}$ is a tangent to the parabola $y^2=4ax$, whatever the value of m. Prove that the point of intersection of any two tangents to a parabola which are perpendicular to each other lies on the directrix.

[42 marks.]

Or,

5. The focus of a parabola is $(1, 2)$ and the equation of the tangent at the vertex is $3x-4y+6=0$. Find the equation of the parabola.

[42 marks.]

6. In a triangle ABC, using the usual notation, prove that

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}.$$

In a triangle ABC the bisector AD of the angle BAC meets BC in D. Show that

$$AD=\frac{2\sqrt{bc}}{b+c}\sqrt{s(s-a)}.$$

[42 marks.]

Or,

6. In a triangle ABC, using the usual notation, prove that

(*a*) (i) $r=\frac{\Delta}{s}$; (ii) $r_1=\frac{\Delta}{s-a}$;

(*b*) $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$;

(*c*) $r_1 r_2 r_3=r^3\cot^2\frac{A}{2}\cot^2\frac{B}{2}\cot^2\frac{C}{2}$.

[42 marks.]

7. (i) Prove that $2\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{5}=\frac{\pi}{4}$;

(ii) Solve the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$;

(iii) Find the general solution of the equation $\sin x+\cos x=\frac{\sqrt{2}}{2}$.

[42 marks.]