AN ROINN OIDEACHAIS

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1949.

MATHEMATICS—Geometry—Honours.

THURSDAY, 9th JUNE.—MORNING, 10 TO 12.30.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. If D, E, F are points on the sides BC, CA, AB respectively of a triangle ABC such that

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1,$$

prove that AD, BE, CF are concurrent.

Find a point O in the plane of a triangle ABC such that

$$\triangle AOB : \triangle BOC : \triangle COA = p : q : r,$$

where p, q, r are given constants. How many solutions are there ?

[40 marks.]

2. If A, B, C, D is a harmonic range and O is the middle point of AC, prove that $OC^2 = OB.OD$.

Two circles intersect at A, B ; a common tangent touches them at P, Q and cuts any other circle through A, B at L, M. Prove that PQ is divided harmonically at L, M.

[40 marks.]

3. Show that the inverse of a circle with respect to a point not on its circumference is another circle.

Show also that a circle, its inverse and the circle of inversion are coaxal with one another.

[40 marks.]

4. Show that the angle between the straight lines

$$y=m_1x+c_1,\ y=m_2x+c_2 \text{ is } \tan^{-1}\frac{m_1-m_2}{1+m_1m_2}.$$

Find the equations of the sides of a square given that the co-ordinates of a pair of opposite vertices are (0, 0) and (1, 2).

[42 marks.]

5. Find the equations of the circles which pass through the points (—1, 1) ; (—4, 4) and touch the straight line $x-y=0$.

[42 marks.]

Or,

5. Show that the circle described on the common chord of the circles $x^2+y^2+2x+8y-4=0$ and $x^2+y^2-x+2y-1=0$ as diameter passes through the origin.

[42 marks.]

6. Find the equation of the parabola whose focus is (2, —3) and vertex (—1, 0).

[42 marks.]

7. Prove that

(i) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$,

(ii) $a\cos A+b\cos B+c\cos C=4R\sin A\sin B\sin C$,

where A, B, C are the angles of a triangle and R is the radius of the circumcircle.

[42 marks.]

Or,

7. In a triangle ABC, $\cos A=-\frac{5}{13}$, $b=13$, $c=4$: find R, r, r_1, r_2, r_3, where these quantities have the usual meanings.

[42 marks.]

8. (*a*) Solve the equations :

(i) $\tan^{-1}\frac{x}{1+x}+\tan^{-1}\frac{x}{1-x}=\tan^{-1}2$,

(ii) $\cos^{-1}\frac{x^2-1}{x^2+1}=\frac{2\pi}{3}-\tan^{-1}\frac{2x}{x^2-1}$.

(*b*) Find the general solution of the equation $6\sin\theta+13\cos\theta=14$.

[42 marks.]