AN ROINN OIDEACHAIS
(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1949.

MATHEMATICS—Algebra—Honours.

TUESDAY, 14*th* *JUNE*.—MORNING, 10 TO 12.30.

Not more than *six* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations

(i) $\sqrt{x^2+6x+1}+\sqrt{x^2+6x-5}=6$;

(ii) $x^2+y^2+x+y=3xy=18$.

[40 marks.]

2. (i) Show that $a+b-c$ is a factor of

$$a^4+b^4+c^4-2a^2b^2-2b^2c^2-2c^2a^2$$

and find the other factors.

(ii) Show that $a(a+b)(a+2b)(a+3b)+b^4$ is a perfect square.

[40 marks.]

3. (*a*) Prove that ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$, where ${}^nC_r$ is the number of combinations of n things taken r at a time.

(*b*) In how many different orders can 6 persons enter a room (i) so that one particular person A shall immediately precede another particular person B, (ii) so that A may precede B but not necessarily immediately.

[40 marks.]

4. (*a*) If the binomial theorem is known to be true for some special positive integral value of the exponent n, say $n=k$, prove that it is true for $n=k+1$.

(*b*) The expansion of $(1+ax)^n$ as far as three terms is

$$1+\tfrac{1}{6}x+\tfrac{1}{24}x^2:$$

find a and n. Find also the fourth term and the $(r+1)$th term.

[40 marks.]

5. Find, to two decimal places, the negative root of

$$x^3-3x^2-13x+20=0.$$

[42 marks.]

Or,

5. Prove that $1+\omega+\omega^2=0$, where ω, ω^2 are the imaginary cube roots of unity.

Find the linear factors of $(a+\omega b)^3+(a+\omega^2 b)^3$.

[42 marks.]

6. (i) Find the nth term and the sum of n terms of the series $11^2+13^2+15^2+\ \cdot\ \cdot\ \cdot$

(ii) The first term of a series is 1 and the nth term exceeds the $(n-1)$th term by n: find the nth term and the sum of n terms.

[42 marks.]

7. (i) Find, from first principles, the derivative of $\tan x$.

(ii) Differentiate (*a*) $\tan^{-1}x$. (*b*) $\tan^{-1}x+\tan^{-1}\frac{1}{x}$.

Account geometrically for your answer in (*b*).

[42 marks.]

8. Trace the curve

$$y=\frac{x^2-4x+1}{x^2+x+1},$$

paying special attention to maximum and minimum points and indicating clearly the position of the curve at large distances from the origin.

[42 marks.]

9. Show that

$$\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

and deduce that $\int_0^{\frac{1}{2}\pi} f(\sin\theta)d\theta=\int_0^{\frac{1}{2}\pi} f(\cos\theta)d\theta.$

Hence, or otherwise, find the values of

(i) $\int_0^{\frac{1}{2}\pi}\sin^2\theta d\theta$, (ii) $\int_0^{\frac{1}{2}\pi}\sin^6\theta d\theta$.

[42 marks.]

Or,

9. Sketch the curve $y^2=(x-1)(x-4)^2$ and find (i) the area of the loop, (ii) the volume generated by the revolution of the loop about the x-axis.

[42 marks.]