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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1948.

MATHEMATICS—Geometry—Honours.

WEDNESDAY, 16*th JUNE*.—Morning, 10 to 12.30.

Six questions may be answered, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. A, B, C, D is a harmonic range and O is any other point. Prove that any transversal is cut harmonically by the lines OA, OB, OC, OD.

If B is the mid point of AC, where is D situated ? In this case, show that one of the rays of the harmonic pencil is parallel to the line containing A, B, C, D.

2. If the polar of a point P, with respect to a given circle, passes through Q, prove that the polar of Q passes through P.

ABC is a triangle and $C > 90°$. O is the orthocentre of ABC and OF is the perpendicular from O on AB. A circle, S, is described with O as centre and radius equal to $\sqrt{(OC.OF)}$. Prove that each side of ABC is the polar of the opposite vertex with respect to the circle S.

3. Given two circles of a non-intersecting coaxal system, show how to construct the radical axis and the limiting points of the system. (Proof not required.)

Prove that any circle passing through the limiting points cuts each circle of the system orthogonally. Use this theorem to construct a circle which cuts each of three given circles orthogonally, where the three given circles are not circles of a coaxal system.

4. If t is a variable parameter, show that the point whose co-ordinates are given by

$$x=\frac{3+t}{2-t}, \quad y=\frac{1-3t}{2-t},$$

lies on a straight line, and find the equation of this line in the form $ax+by+c=0$.

Show that the point given by

$$x=1+\lambda, \quad y=1-\lambda,$$

where λ is a variable parameter, lies on the same straight line.

Find the value of λ which yields the same point as that given by $t=t_1$.

5. The point P_1, whose coordinates are (x_1, y_1), is joined to the centre, O, of the circle $x^2+y^2=r^2$. A point P_2 is taken on OP_1 such that $OP_1 . OP_2=r^2$. If the co-ordinates of P_2 are (x_2, y_2), show that

$$x_2=\frac{r^2x_1}{x_1{}^2+y_1{}^2}, \quad y_2=\frac{r^2y_1}{x_1{}^2+y_1{}^2}.$$

Hence, or otherwise, find the equation of the inverse of the line $3x+4y+5=0$, with respect to the circle $x^2+y^2=r^2$. Show that the inverse is a circle of radius $\frac{1}{2}r^2$.

6. Find the equation of the tangent at the point $(at^2, 2at)$ to the parabola $y^2=4ax$.

Tangents are drawn to the parabola at two points P and Q. These intersect at the point A, and PQ cuts the axis at B; prove that the distance of B from the focus is equal to the distance of A from the directrix.

SECTION B.

7. (*a*) Find the general solutions of the equation

$$\sin 2\theta+\sin 3\theta=\sin\theta.$$

(*b*) Prove that the statement

$$\sin(\alpha+\beta)=\sin\alpha+\sin\beta$$

is not true unless one of the angles, α, β, $(\alpha+\beta)$, is equal to $2n\pi$ where n is zero or an integer.

8. With the usual notation, prove that in a triangle ABC,

(i) $\dfrac{1}{r_2}+\dfrac{1}{r_3}=\dfrac{a}{r_1(s-a)}$,

(ii) $\dfrac{2}{p_1}=\dfrac{1}{r_2}+\dfrac{1}{r_3}$,

(iii) $\dfrac{1}{p_1}+\dfrac{1}{p_2}+\dfrac{1}{p_3}=\dfrac{1}{r}$,

where p_1, p_2, p_3, are the perpendiculars from A, B, C respectively, on the opposite sides.

9. If a, b, c be real numbers such that $a^2+b^2>c^2$ and if α, β are distinct values of θ, whose difference is not a multiple of 2π, which satisfy the equation

$$a\cos\theta+b\sin\theta=c,$$

prove that

(i) $\tan\frac{1}{2}\alpha+\tan\frac{1}{2}\beta=\dfrac{2b}{c+a}$,

(ii) $\tan\frac{1}{2}\alpha\tan\frac{1}{2}\beta=\dfrac{c-a}{c+a}$.

Deduce the value of $\tan\frac{1}{2}(\alpha+\beta)$ and show that

$$\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2}.$$

Or,

Prove that the limit of $\sin\theta\div\theta$, as θ tends to zero, is equal to 1.

Find the area of a regular polygon of n sides inscribed in a circle of radius r, and, by making n go to infinity, deduce that the area of the circle is equal to πr^2.

Use the same method to show that the length of the circumference of the circle is $2\pi r$.