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LEAVING CERTIFICATE EXAMINATION, 1948.

MATHEMATICS—Algebra—Honours.

MONDAY, 21*st JUNE*.—MORNING, 10 TO 12.30.

Six questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. Solve the simultaneous equations

$$y^2+yz+z^2=1,$$
$$z^2+zx+x^2=4,$$
$$x^2+xy+y^2=7.$$

[40 marks.]

2. Prove that the sum of the cubes of the first n natural numbers is $\frac{1}{4}n^2(n+1)^2$.

Find the sum of the series

$$1^3-2^3+3^3-4^3+\quad . \quad . \quad . \quad . \quad -(2n)^3+(2n+1)^3.$$

[40 marks.]

3. A certain examination consists of 3 mathematical papers and 7 non-mathematical papers. In how many different orders can the papers be set?

In how many different orders can the papers be set if

(i) the mathematical papers are not *all* to be consecutive,

(ii) no two mathematical papers are to be consecutive?

[40 marks.]

4. If $x+y+z=0$, prove that

(i) $x^3+y^3+z^3=3xyz$,

(ii) $\frac{1}{5}(x^5+y^5+z^5)=\frac{1}{2}xyz(x^2+y^2+z^2)$.

[40 marks.]

5. Find the values of A, B and C if

$$\frac{x+1}{(2+x)(1-x)^2}=\frac{A}{1-x}+\frac{B}{(1-x)^2}+\frac{C}{2+x}$$

where A, B and C are independent of x.

Hence, or otherwise, find the first three terms and the coefficient of x^n in the expansion of

$$\frac{x+1}{(2+x)(1-x)^2}$$

in ascending powers of x.

[42 marks.]

6. If a_1, a_2, a_3 are real positive quantities prove that

(i) $(a_1+a_2)(a_1^{-1}+a_2^{-1})\geqslant 4$.

(ii) $(a_1+a_2+a_3)(a_1^{-1}+a_2^{-1}+a_3^{-1})\geqslant 9$.

[42 marks.]

SECTION B.

7. (i) Differentiate, with respect to x, the functions

$$\frac{\sin x+\cos x}{\sin x-\cos x};\ \cot[\sqrt{1-x}]$$

(ii) If $y=x^{-n}\sin x$ prove that

$$x^2\frac{d^2y}{dx^2}+2nx\frac{dy}{dx}+[n(n-1)+x^2]y=0.$$

[42 marks.]

8. Evaluate:

(i) $\int_0^{\frac{1}{2}\pi}\sin^3x\cos^2x\,dx$; (ii) $\int_0^{\frac{1}{3}\pi}\frac{\sin^3x}{\cos^2x}\,dx$; (iii) $\int_0^2\sqrt{4-x^2}\,dx$.

[In (iii) put $x=2\sin\theta$.]

[42 marks.]

9. A conical tank stands with its axis vertical and apex down. It is 10 ft. deep and its top is 8 ft. in diameter. Water is flowing into the tank at a uniform rate of 16 cu. ft. per minute. Find the rate at which (i) the depth, (ii) the area of the free surface of the water, is increasing when the water in the tank is 4 ft. deep.

[42 marks.]

10. Find the slope of the tangent to the curve $x=a(t-\sin t)$, $y=a(1-\cos t)$ at the point at which the parameter $t=t_1$.

Prove that the area enclosed by the x-axis and the portion of the curve for which $0\leqslant t\leqslant 2\pi$ is $3\pi a^2$.

Or

In the case of the curve

$$y^2-4xy+x^3=0$$

(i) find the range of values of x for which real points on the curve exist; (ii) find the points at which the tangent to the curve is parallel to the x-axis and y-axis respectively; (iii) trace the curve.

[42 marks.]