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LEAVING CERTIFICATE EXAMINATION, 1946.

MATHEMATICS—Geometry—Honours.

THURSDAY, 13*th* *JUNE*.—Afternoon, 3 to 5.30.

Six questions may be answered, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. Define Harmonic Range.

If AB is divided harmonically at C and D, prove that DC is divided harmonically at B and A.

ABCD is a quadrilateral; AC, BD meet at O; AD, BC produced meet at P; AB, DC produced meet at Q; PO produced meets AB at X. Show that AB is divided harmonically at X and Q.

[Hint: Apply Ceva's and Menelaus' theorems to the triangle PAB.]

2. Write a short account of coaxial circles including (i) definition, (ii) limiting points, (iii) any fundamental theorem associated with coaxial circles. Give proof of the theorem you mention.

3. Two tangents are drawn to a circle from a point P which is on the polar of a point Q. Prove that the two tangents from P, the polar of Q, and the line PQ form a harmonic pencil. (You may assume the harmonic property of pole and polar.)

Use this theorem to prove the following:—

The incircle of a triangle ABC touches the sides BC, CA, AB at X, Y, Z respectively. If ZY produced meets BC produced at S, show that X and S divide BC harmonically.

4. Find the coordinates of a point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ratio $l : m$. Examine the case for internal section and *state* the result for external section.

The points (—1, —1), (1, 3), (3, 1) are three vertices of a parallelogram; the fourth vertex lies in the fourth quadrant. Find the coordinates (i) of the point of intersection of the diagonals, and (ii) of the fourth vertex.

5. Find the coordinates of the centre and the length of the radius of the circle

$$x^2+y^2-2x-4y+4=0.$$

Find the values of m for which the line $x=my$ touches the given circle. Hence find the equations of the tangents from the origin to the circle.

6. P is any point on the parabola $y^2=4ax$, whose focus is F. The point N is the foot of the perpendicular from P on the x-axis, and G is the point in which the normal at P meets the x-axis. Prove that

(i) angle FPG=FGP,

(ii) the length of NG is constant,

(iii) the foot of the perpendicular from F on the tangent at P lies on the y-axis.

Section B.

7. If $A+B+C=180^\circ$, prove that

(i) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$,

(ii) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$,

(iii) $2-(\sin^2A+\sin^2B+\sin^2C)=-2\cos A\cos B\cos C$.

8. (*a*) Find the general solution of the equation

$$\cos 3\theta-\cos 4\theta+\cos 5\theta=0.$$

(*b*) Solve generally for x and y

$$2\cos(x-y)=\cos(x+y)=1.$$

9. Express $a\sec x+b\tan x$ in terms of $\tan\frac{1}{2}x(=t)$.

If α, β are two distinct solutions of the equation

$$a\sec x+b\tan x=c,$$

prove that

$$\cos(\alpha-\beta)=\frac{2a^2-b^2-c^2}{b^2+c^2}.$$

10. (*a*) Prove that

$$\operatorname{cosec}2\theta=\cot\theta-\cot 2\theta.$$

Hence find the sum of n terms of the series

$$\operatorname{cosec}2\theta+\operatorname{cosec}4\theta+\operatorname{cosec}8\theta+\ldots.$$

(*b*) Prove that

$$\tan\theta=\cot\theta-2\cot 2\theta.$$

Hence find the sum of n terms of the series

$$\tan\theta+\tfrac{1}{2}\tan\tfrac{1}{2}\theta+\tfrac{1}{4}\tan\tfrac{1}{4}\theta+\ldots.$$