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(Department of Education.)

LEAVING CERTIFICATE EXAMINATION, 1946.

MATHEMATICS—Algebra—Honours.

WEDNESDAY, 12th JUNE—Morning, 10 to 12.30.

Six questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent

Section A.

1. Find *all* the values of x and y which satisfy the equations

$$x^2+y^2=61$$
$$x^3-y^3=91.$$

[Hint: Put $u=x-y$, $v=xy$.]

[40 marks.]

2. If $x^3+y^3+z^3=3xyz$, and x, y and z are real quantities, prove that either $x+y+z=0$ or $x=y=z$.

Factorise

$$(a^2-bc)^3(b-c)^3+(b^2-ca)^3(c-a)^3+(c^2-ab)^3(a-b)^3.$$

[40 marks.]

3. Find the nth term in the series

$$\frac{1}{1.3}+\frac{2}{1.3.5}+\frac{3}{1.3.5.7}+\frac{4}{1.3.5.7.9}+\ .\ .\ .\ .\ ,$$

and show that it may be written in the form

$$\frac{A}{1.3.5.\ .\ .\ .(2n-1)}-\frac{B}{1.3.5.\ .\ .\ .(2n-1)(2n+1)}$$

where A and B are independent of n.

Find the sum of the series to n terms and to infinity.

[40 marks.]

4. A committee of six is to be chosen from nine men and three women. In how many ways may this be done

(*a*) if all the twelve are equally eligible for election,

(*b*) if the committee is to contain one woman only,

(*c*) if the committee is to contain at least one woman ?

[40 marks.]

5. Write down the first four terms in the expansion, in ascending powers of x, of $(1+\frac{1}{2}x)^{-2}$. From it deduce the expansion of $(2+y+y^2)^{-2}$ in ascending powers of y as far as the term containing y^3.

Find the first three terms in the expansion of

$$\frac{1-y}{(2+y+y^2)^2}$$

[42 marks-]

6. If ω is one of the imaginary cube roots of unity show that the other is ω^2 and prove that

$$1+\omega+\omega^2=0.$$

If

$$x=a+b, \quad y=a\omega+b\omega^2, \quad z=a\omega^2+b\omega$$

show that

$$x^2+y^2+z^2=6ab$$

and

$$x^3+y^3+z^3=3(a^3+b^3).$$

[42 marks.]

SECTION B.

7. (i) Differentiate with respect to x the functions

$$(a)\ \frac{1-\sqrt{x}}{1+\sqrt{x}}, \quad (b)\ \sqrt{1-x^2}.\sin 2x.$$

(ii) If $y=f(u)$ and $u=F(x)$ explain clearly why we may write

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}.$$

If $y=\sin^{-1}u$ and $u=\dfrac{x}{\sqrt{1+x^2}}$, find dy/dx in terms of x.

[42 marks.]

8. A vertical cylindrical boiler has a flat base and a hemispherical top. If the length of the cylindrical part is h ft. and the radius of the top is r ft., find the ratio of r to h so that the volume may be a maximum for a given total surface area.

[42 marks.]

9. Evaluate

$$\text{(i)} \int_0^{\frac{\pi}{8}} \sec^2 2x \tan 2x\,dx, \quad \text{(ii)} \int_0^{\frac{\pi}{2}} \sin 3x \cos 2x\,dx, \quad \text{(iii)} \int_6^{11} x\sqrt{x-2}\,dx.$$

[42 marks.]

10. Show that the area between the parabola $y^2=4ax$ and the tangents at the ends of the latus rectum is $4a^2/3$.

Find the volume generated by the revolution of this area round the x-axis.

[42 marks.]

11. Sketch the curves

$$y=x^2(x-2) \text{ and } y=x^3(x-2)$$

between $x=-1$ and $x=2\frac{1}{2}$, paying particular attention to the portions in the neighbourhood of the origin.

Determine the points of maximum and minimum and the points of inflexion on the curves.

[42 marks.]