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LEAVING CERTIFICATE EXAMINATION, 1945.

MATHEMATICS—Geometry—Honours.

WEDNESDAY, 13th JUNE.—AFTERNOON, 3 TO 5.30.

Six questions may be answered, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. Find the locus of the vertex of a triangle when the base is given in magnitude and position and the ratio of the other two sides is known.

What does the locus become if the given ratio is one of equality?

2. $ABCD$ is a quadrilateral and AD, BC, produced, meet at P, while DC, AB, produced, meet at Q. X is the point of intersection of AC, BD; AC produced meets PQ at Y: prove that $AXCY$ is a harmonic range.

3. Show that the inverse of a circle with respect to a point not on its circumference is a circle.

A circle of centre C is inverted with respect to a point O, not on its circumference. Show that the inverse of the point C is the point at which OC cuts the polar of O with respect to the inverse circle.

4. Show that the triangle whose vertices are the points (1, 2), (2, 3), (3, 1) is isosceles.

Calculate the co-ordinates of (i) the centroid, (ii) the orthocentre.

5. Find the equation of the tangent at the point $(a\cos\theta, a\sin\theta)$ on the circle $x^2+y^2=a^2$.

Tangents are drawn from the point $(2a, 2a)$ to the circle $x^2+y^2=a^2$. If the coordinates of a point of contact be denoted by $(a\cos\theta, a\sin\theta)$, prove that $\tan\frac{1}{2}\theta=\frac{1}{3}(2\pm\sqrt{7})$.

6. Show that the straight line

$$x-ty+at^2=0$$

touches the parabola $y^2=4ax$, for all values of t.

Find the coordinates of the point of contact and prove that the normal at the point of contact meets the parabola again at the point

$$\left\{\frac{a(t^2+2)^2}{t^2}, \frac{-2a(t^2+2)}{t}\right\}.$$

SECTION B.

7. ABC is a triangle in which $b > c$. The internal and external bisectors of the angle A meet BC in X and Y respectively; prove that

$$\text{(i)}\ AX = \frac{2bc}{b+c}\cos\tfrac{1}{2}A,$$

$$\text{(ii)}\ AY = \frac{2bc}{b-c}\sin\tfrac{1}{2}A.$$

8. Factorize

$$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma).$$

Hence, or otherwise, show that

$$\sin A + \sin B + \sin C = 4\cos\tfrac{1}{2}A\cos\tfrac{1}{2}B\cos\tfrac{1}{2}C,$$

where A, B, C are the angles of a triangle.

9. (*a*) Find the general solution of the equation

$$\cos 7\theta = \cos 5\theta.$$

(*b*) Solve the equation

$$4\cos\theta = \operatorname{cosec}\theta.$$

Show that the general solution can be put in the form

$$\theta = k\pi + \pi/12 \text{ or } k\pi + 5\pi/12,$$

where k is an integer.

10. ABC is an acute-angled triangle in which $C > B$. If the incentre be denoted by I and the circumcentre by O, prove that

(i) $IA = 4R\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C$,

(ii) angle $OAI = \tfrac{1}{2}(C-B)$,

(iii) $OI^2 = R^2 - 2Rr$.