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LEAVING CERTIFICATE EXAMINATION, 1945.

MATHEMATICS—Algebra—Honours.

TUESDAY 12*th* *JUNE*—Morning, 10 to 12.30.

Six questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. Find the real values of x and y which satisfy the equations:

$$2x^3+9x^2y-2y^3+40=0$$
$$x^3+x^2y-xy^2+8=0.$$

[40 marks.]

2. Show that the nth term of the series

$$\frac{2^2}{1(1+2)}+\frac{3^2}{(1+2)(1+2+3)}+\frac{4^2}{(1+2+3)(1+2+3+4)}+\ldots\ldots$$

may be written in the form $2/n-2/(n+2)$.

Hence, or otherwise, show that the sum of the series, for all positive integral values of n, is $n(3n+5)/(n+1)(n+2)$. What is the sum to infinity of the series? [40 marks.]

3. Show that the equation

$$x^3+x^2-3=0$$

has a root between 1 and 2.

By substituting $x=1+u$ in the equation and retaining only the first power of u verify that $x=1{\cdot}2$ is an approximation to this root. Then substitute $x=1{\cdot}2+v$ in the equation, neglect the square and cube of v, and obtain a better approximation. Continue this process and find the root of the equation correct to three places of decimals. [40 marks.]

4. How many four-figure numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 when (i) it is not permitted, (ii) it is permitted, to use any digit more than once in the same number?

In the second case in how many numbers are there (a) four digits alike, (b) three digits alike and the fourth unlike? [40 marks.]

5. Write down the first four terms in the expansion of (i) the nth root of $1+x$ and (ii)

$$\frac{2n+(n+1)x}{2n+(n-1)x},$$

in ascending powers of x. Show that the fraction is an approximate value of the root when x is small and $n>1$.

Use the above result to write down a rational fraction which is an approximation to $\sqrt[5]{3150}$. [42 marks.]

6. (i) Prove that the arithmetic mean between two positive numbers is greater than their geometric mean and deduce that if x, y and z are positive numbers

$$(x+y)\ (y+z)\ (z+x)>8xyz.$$

(ii) If a and b are positive proper fractions prove that

(a) $1-a<\dfrac{1}{1+a}$.

(b) $1-a-b<(1-a)\ (1-b)<\dfrac{1}{(1+a)\ (1+b)}<\dfrac{1}{1+a+b}$.

[42 marks.]

Section B.

7. (i) If $y=x^{-2}$ find, from first principles, the value of dy/dx.

(ii) Find the value of dy/dx if

(a) $y=(1+x)^{\frac{1}{2}}/(1-x)$, (b) $y=\tan^{-1}\sqrt{x}$.

[42 marks.]

8. A ploughed field is in the form of a square $ABCD$ each side being 150 yards long. A pathway runs along one side AB only. A man can run at the rate of 340 yards a minute along the path and at the rate of 160 yards a minute over the ploughed ground. Find his quickest route from the corner A to the opposite corner C.

[42 marks.]

9. Evaluate :

(i) $\displaystyle\int_0^{\pi/4} \sin^2 2x\, dx$, (ii) $\displaystyle\int_1^8 \frac{(x+1)\ dx}{\sqrt[3]{x}}$,

(iii) $\displaystyle\int_1^2 \frac{dx}{\sqrt{4x-x^2}}$.

In (iii) put $x=2(1-\cos\theta)$.

[42 marks.]

10. The curve $y^2=x-1$ cuts the x-axis at A and cuts the curve $y^2=4(x-4)$ at B and D. The curve $y^2=4(x-4)$ cuts the x-axis at C. Find the area $ABCD$ enclosed between the two curves.

Find the volume generated by the revolution of this area around the line AC.

[42 marks.]

11. The curve

$$y^2=ax^3+bx^2+cx+d$$

passes through the origin and through the points (1,1), (4,4) and (9, 21). Determine the values of the coefficients a, b, c and d.

Draw a rough graph of the curve and indicate on it the points at which the tangent to the curve is parallel to the x-axis.

[42 marks.]