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(Department of Education.)

LEAVING CERTIFICATE EXAMINATION, 1944.

MATHEMATICS—Geometry—Honours.

TUESDAY, 13*th JUNE*.—Afternoon 3 to 5.30.

Seven questions to be attempted and not more than *five* from Section A.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. A straight line cuts the sides BC, CA, AB of a triangle ABC in L, M, N respectively; using a certain convention of signs, prove that

$$\frac{BL}{LC}\cdot\frac{CM}{MA}\cdot\frac{AN}{NB}=-1.$$

If A is joined to the intersection of BM and CN and the joining line meets BC at D, prove that L and D are harmonic conjugates with respect to B and C.

2. If a straight line is drawn through any point to cut a circle, prove that the line is cut harmonically by the circle, the point and the polar of the point.

Two tangents are drawn from a point P on the polar of a point Q; prove that the two tangents from P, the polar of Q and the line PQ form a harmonic pencil.

3. Prove that the angle between any two curves is equal to the angle between their inverses.

Prove that a coaxal system of non-intersecting circles can be inverted into a system of concentric circles by taking one of the limiting points as centre of inversion.

[Hint. Use: Any circle passing through the limiting points of a coaxal system cuts every circle of the system orthogonally.]

4. Find the coordinates of the centroid and of the orthocentre of the triangle whose sides are

$$2x-y+2=0,\ 3x+y-3=0,\ y+1=0.$$

5. Prove that the circles

$$x^2+y^2+2gx+2fy+c=0,\ x^2+y^2+2g'x+2f'y+c'=0$$

cut orthogonally if $2(gg'+ff')=c+c'$.

[Use: $r^2+r_1^2=d^2$, where r, r_1 are the radii of the circles and d is the distance between their centres.]

Find the equation of the circle passing through the points (1, 1), (2, 3) and cutting orthogonally the circle

$$x^2+y^2-8x-2y+16=0.$$

6. If $S \equiv x^2+y^2-a^2$, $L \equiv x\cos\alpha+y\sin\alpha-p$, interpret the equation $S+\lambda L=0$, where λ is a constant.

$S=0$ cuts $L=0$ in the points A, B and a circle is described on AB as diameter. Prove that its equation is $S-2pL=0$.

SECTION B.

7. Defining a parabola by its focus-directrix property, prove that its equation can be got in the form $y^2=4ax$.

P is any point on a parabola whose focus is F; PM is the perpendicular on the directrix and T is the point where the tangent at P meets the axis of the parabola; PN is the perpendicular from P on the axis of the parabola, and G is the point where the normal at P meets the axis. Prove that

(i) NG is constant for all positions of P, on the parabola,

(ii) FT=FP,

(iii) PT bisects the angle MPF.

8. (*a*) With the usual notation for a triangle ABC, prove that

(i) $r=4R\sin\frac{1}{2}A\sin\frac{1}{2}B\sin\frac{1}{2}C$,

(ii) $s=4R\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C$.

(*b*) Show that the triangle ABC is right angled when $r_1=r+r_2+r_3$.

9. Show how to express the equation $a\cos\theta+b\sin\theta=0$ as a quadratic in t, where $t=\tan\frac{1}{2}\theta$.

If α, β, whose difference is not a multiple of 2π, are roots of the equation in θ

$$a\cos\theta+b\sin\theta=c,$$

prove that

$$\tan\tfrac{1}{2}(\alpha+\beta)=b/a.$$

Hence, or otherwise, show that

(i) $\cos(\alpha+\beta)=\dfrac{a^2-b^2}{a^2+b^2}$,

(ii) $\sin(\alpha+\beta)=\dfrac{2ab}{a^2+b^2}$.

10. (*a*) Find general solutions of the equation,

$$\sin4\theta-\sin2\theta=\cos3\theta.$$

(*b*) Find the ranges of values of θ for which the equation in x

$$x+1/x=4\cos\theta$$

has real roots.