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(Department of Education).

## LEAVING CERTIFICATE EXAMINATION, 1944.

## MATHEMATICS—Algebra—Honours.

*WEDNESDAY*, 14*th* *JUNE*.—MORNING, 10 TO 12.30.

*Six* questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent.

### SECTION A.

1. Solve the simultaneous equations:

$$x + y + z = 5.$$
$$1/x + 1/y + 1/z = 1/5.$$
$$yz + zx + xy = -9.$$

[40 marks].

2. (i) Show that:

$$\sum_{r=1}^{n} r(r+1) = 1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \tfrac{1}{3}n(n+1)(n+2).$$

Prove any formula used in the question.

(ii) Show that the $n$th term of the series

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \frac{1}{10.13} + \ldots$$

may be expressed in the form $\frac{1}{3}\left[\frac{1}{3n-2} - \frac{1}{3n+1}\right]$, and find the sum of the series to $n$ terms and to infinity.

[40 marks.]

3. There are five seats on either side of a railway compartment. In how many different ways can ten persons take their seats in the compartment?

If two of the persons refuse to sit with their backs to the engine in how many ways can the ten persons be seated in the compartment?

[40 marks.]

4. Write down, in ascending powers of $x$, the first four terms in the expansion of (i) $(1 + \frac{1}{2}x)^{-2}$, (ii) $(1-3x)^{\frac{1}{3}}$. May $(1 + \frac{1}{2}x)^{-2}$ be expanded as an infinite series if $x = 3$?

Find the first three terms in the expansion of

$$\frac{(1-3x)^{\frac{1}{3}}}{(2+x)^2}$$

in ascending powers of $x$.

[40 marks.]

5. (i) If $x$, $y$ and $m$ are real numbers, and $x > y$, under what conditions will the following inequalities be true (*a*) $mx > my$ ; (*b*) $x^2 > y^2$ ; (*c*) $1/x < 1/y$ ?

(ii) If $z$ is real for what values of $z$ is $1-z > \frac{1}{1+z}$ ?

[42 marks.]

6. (i) Solve the quadratic equation $x^2 + x + i\sqrt{3} = 0$, expressing the roots in the form $a + ib$, where $a$ and $b$ are real quantities and $i^2 = -1$.

(ii) Express $(x + iy)^3$ in the form $a + ib$ and deduce an expression for $(x^2 + y^2)^3$ as the sum of two squares.

[42 marks.]

SECTION B.

7. Prove that, if $\theta$ is a positive acute angle less than $\frac{1}{2}\pi$,

$$\sin\theta < \theta < \tan\theta.$$

Find from first principles the differential coefficient of $\cos x$ with respect to $x$.

If $y = \cos x^\circ$ write down the value of $dy/dx$.

[42 marks.]

8. (i) Differentiate, with respect to $x$, the functions :—

$$\sqrt{x(x+1)}, \qquad \frac{2x+1}{\sqrt{x(x+1)}}.$$

(ii) If $x = 2at^3$, $y = 3at^2$ express $dy/dx$ and $d^2y/dx^2$ in terms of $t$.

[42 marks.]

9. Evaluate

$$\text{(i)} \int_0^{\frac{\pi}{2}} \frac{\sin x\,dx}{(3+2\cos x)^2}, \qquad \text{(ii)} \int_1^5 x(x-1)^{\frac{1}{2}}dx,$$

$$\text{(iii)} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\,dx$$

[42 marks.]

10. A curve of the form $y = a + bx + cx^2$ is drawn through the points $(-h, y_1)$, $(0, y_2)$, $(h, y_3)$. Find the values of the coefficients $a$, $b$ and $c$ in terms of $y_1$, $y_2$, $y_3$ and $h$.

Show that the area enclosed between the curve, the $x$-axis and the ordinates $x = \pm h$ is $\frac{1}{3}h(y_1 + 4y_2 + y_3)$.

[42 marks.]

11. If $f(x) \equiv x(x+a)(x+2)^2$ is a function of $x$ such that $f'(x) = 0$ when $x = -\frac{1}{2}$ determine $a$.

With this value of $a$ trace the curve $y = f(x)$ noting the turning points, points of inflexion and infinite branches.

[42 marks.]