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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1943.

MATHEMATICS—Algebra—Honours.

WEDNESDAY, 9th JUNE.—MORNING, 10 TO 12.30.

Six questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1\. Solve the equations :

$$x^2y+xy^2+x+y=3$$
$$2(x-1)(y-1)+1=0.$$

[40 marks.]

2\. (i) Factorise

$$(x+y+z)^5-x^5-y^5-z^5.$$

(ii) From the factors of

$$x^2(y-z)+y^2(z-x)+z^2(x-y)$$

deduce, by making suitable substitutions for x, y and z, the factors of

$$(b-c)(b+c-2a)^2+(c-a)(c+a-2b)^2+(a-b)(a+b-2c)^2.$$

[40 marks.]

3\. The natural numbers are divided into groups thus :

$$1+(2+3+4)+(5+6+7+8+9)+(10+11+12+13+14+15+16) + \ldots$$

Find the total of the numbers in the first n groups and the sum of the numbers in the nth group.

[40 marks.]

4\. Two families each consist of 5 brothers and 5 sisters. In how many different ways may a party of 8 persons, consisting of 4 men and 4 women, be made up from these families if four members of each family are to be included ?

[40 marks.]

5\. If $a=b(1+x)$, where x is a small quantity, expand $\sqrt{(a/b)}$ and $a/(a+b)$ in ascending powers of x as far as the term containing x^4.

Show that, neglecting the fourth and higher powers of x,

$$\sqrt{\frac{a}{b}}=\frac{a}{a+b}+\frac{1}{4}\,\frac{a+b}{b}.$$

[42 marks.]

6. (i) If a, b, c and d are real numbers and $i=\sqrt{-1}$, show that the condition that the equation

$$z^2+(a+ib)z+(c+id)=0$$

may have one real root is $d^2+b^2c=abd$.

(ii) If $z=1-i\sqrt{3}$, where $i=\sqrt{-1}$, express

$$z(2-z),\ z^{-1},\ z^{\frac{1}{2}}$$

in the form $x+iy$, x and y being real numbers, positive or negative.

[42 marks.]

SECTION B.

7. Differentiate with respect to x the functions :

$$\text{(i)}\ \frac{1-x}{1+x+x^2}\ ;\quad \text{(ii)}\ \sin 2x+\cos^2 x.$$

If $y=x^{-1}\sin x$ show that

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0.$$

[42 marks.]

8. An open rectangular tank is to have a volume of 3,888 cubic feet and the edges of the base are to be in the ratio 3 : 1. Show that the cost of lining it with lead is least when its depth is 9 feet.

[42 marks.]

9. Evaluate

$$\text{(i)}\ \int_0^1 \frac{x\,dx}{(1+x^2)^3}\ ;\quad \text{(ii)}\ \int_0^{\frac{\pi}{4}} \tan^2 x\,dx\ ;$$

$$\text{(iii)}\ \int_0^{\frac{\pi}{2}} \sin^3 x\cos^2 x\,dx.$$

[42 marks.]

10. Find the area enclosed by the curves

$$2y=x^3 \text{ and } y^2=8x.$$

Find, also, the volume generated by revolving this area round the x—axis.

[42 marks.]

11. Trace the curve

$$y^2=x(x-3)^2.$$

Show that the curve lies entirely on one side of one of the axes and that it is symmetrical with respect to the other axis. Where are its turning points ?

[42 marks.]