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(Department of Education.)

LEAVING CERTIFICATE EXAMINATION, 1942.

MATHEMATICS— Geometry— Honours.

WEDNESDAY, 10th *JUNE*—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Define pole and polar. What are conjugate points ?
   If P, R are conjugate points with respect to a circle and if PR meets the circle at H, K, prove that (PR ; HK) is harmonic.

   [40 marks.]

2. Show that the inverse of a circle with respect to any point on it is a straight line.
   A system of circles intersect at A, B. What is the inverse of the system with respect to A ?

   [40 marks.]

3. If D is a point on the base BC of a triangle ABC such that mBD$=n$DC, prove that
   $$m\text{AB}^2+n\text{AC}^2=(m+n)\text{AD}^2+m\text{DB}^2+n\text{DC}^2.$$
   Find the position of a point P in a given triangle ABC such that PA^2+PB^2+PC^2 has its least value. Give proof.

   [40 marks.]

4. Prove that $\tan^{-1}\dfrac{a}{b}+\tan^{-1}\dfrac{b-a}{a+b}=\dfrac{\pi}{4}$. Verify the identity by means of a geometrical diagram when $b>a$.

   [40 marks.]

5. Prove the formula for the area of a triangle in terms of the sides.
   The sides of a triangle are $6-\sqrt{2}$; 6; $6+\sqrt{2}$ inches long respectively.
   Find the area of the triangle and the radius of the circumcircle.

   [40 marks.]

6. Find the general solutions of the equations :—
   (i) $12\sin\theta+35\cos\theta=18\frac{1}{2}$
   (ii) $\tan x+\sec 2x=1$.

   [42 marks.]

7. The four points A, B, C, D form an harmonic range. The co-ordinates of A, B, C are $(-4, -2)$; $(3\frac{1}{2}, 6\frac{3}{4})$; $(8, 12)$ respectively. Find the co-ordinates of D.

[42 marks.]

8. Find the equation of the circle which passes through the points of intersection of $x^2+y^2+3x-1=0$ and $3x+4y-4=0$ and through the point $(1, 2)$.

Find the equation of the tangent to the circle at the point $(1, 2)$.

[42 marks.]

9. Show that $x^2+y^2+2\lambda x+c^2=0$ represents a system of coaxal circles for all values of λ and find the co-ordinates of the limiting points. Find also the equation of the orthogonal system of coaxal circles.

[42 marks.]

10. Find the equation of the parabola whose focus is $(4, -2)$ and directrix $y=x+5$.

Find also the co-ordinates of its vertex and the co-ordinates of the extremities of the latus rectum.

[42 marks.]