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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1942.

MATHEMATICS—Algebra—Honours.

MONDAY, 15th JUNE.—AFTERNOON, 3 TO 5.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Find the real and imaginary values of x, y that satisfy the equations

$$\sqrt{x}+\sqrt{y}=3\,;\quad x^2+y^2=17.$$

[40 marks.]

2. (i) If $z^2=x^2+y^2$, express in its simplest form
$$\sqrt{(x+y+z)(x+y-z)(z+x-y)(y+z-x)}.$$

(ii) If $\sqrt{x+a+b}+\sqrt{x+c+d}=\sqrt{x+a-c}+\sqrt{x-b+d}$, prove that $b+c=0$.

[40 marks.]

3. Sum to n terms the series :—

(i) $1^2+2^2+3^2+\ \ldots$

(ii) $1.2+3.5+5.8+7.11+\ \ldots$

[40 marks.]

4. Find, to one decimal place, the real root of $x^3-200x-5000=0$.
[40 marks.]

5. Show that the number of selections that can be made from n unlike things, taking any number at a time, is 2^n-1.

How many different sums of money can be made up from a pound note, a ten-shilling note, a half-crown, three shillings and four pennies ?
[40 marks.]

6. Show that

$$\sqrt{\frac{a+x}{a-x}}=\left(1+\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{-\frac{1}{2}}.$$

Expand the expression by the Binomial Theorem and use the expansion to find the value of $\sqrt{2}$ to 4 decimal places.

[42 marks.]

7. Find from *first principles* the differential coefficient of x^3-3x^2+x-4 with respect to x.

Differentiate $\sqrt{x}\sin^2 3x$.

[42 marks.]

8. Find the volume of the largest lidless box, whose base is a square, that can be made from 100 squares inches of tin.

[42 marks.]

9. Find the value of (i) $\int_0^a y^2dx$ and (ii) $\int_0^a ydx$, where $x^2+y^2=a^2$.

Use these integrals to verify the formulae for the volume of a sphere and the area of a circle respectively.

[42 marks.]

10. Trace the curve $y=x^3(x-4)$, noting turning points, points of inflexion, and infinite branches.

Find the area bounded by the axis of x and that part of the curve which lies below the axis of x.

[42 marks.]