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(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
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LEAVING CERTIFICATE EXAMINATION, 1941.

HONOURS.

MATHEMATICS
(Algebra).

TUESDAY, 17th JUNE.—AFTERNOON, 3 TO 5.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the equations
$x^2-5x(y+9)+2(y+9)^2=2,$
$x^2+x(y+9)-10(y+9)^2=20.$ [40 marks.]

2. (i) If $x+y+z=0$, prove that
$x^4+y^4+z^4=2(x^2y^2+y^2z^2+z^2x^2).$
(ii) Factorise
$a^2(a-1)(b-c)+b^2(b-1)(c-a)+c^2(c-1)(a-b).$
[40 marks.]

3. (i) Find the sum of n terms of
$(n+1)^2+(n+2)^2+(n+3)^2+\ldots\ldots\ldots$
(ii) If $f(n)=\frac{1}{n^2}$, show that
$$f(n)-f(n+1)=\frac{2n+1}{n^2(n+1)^2}.$$
Hence find the sum of n terms of
$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\ldots\ldots\ldots$$
[40 marks.]

4. In a pack of cards there are 52 different cards made up of 4 suits of 13 cards each. In how many ways can 3 cards be selected from the pack? In how many of these ways will the three cards be (i) all of the same suit, (ii) all of different suits?

[40 marks.]

5. Find the greatest root of the equation

$$2x^3-4x+1=0,$$

correct to 2 decimal places. [40 marks.]

6 Expand $(1+\frac{4}{5}x)^{12}$ by the Binomial Theorem. Find (i) the value of the greatest coefficient, (ii) the value of the greatest term when $x=\frac{2}{3}$.

[42 marks.]

7. (*a*) Prove from first principles that

$$\frac{d}{dx}(x\sin x)=\sin x+x\cos x.$$

(*b*) Differentiate (i) $\dfrac{3x-2}{2x+3}$; (ii) $\tan^2(5x+1)$.

[42 marks.]

8. Find the value of

(i) $\int_0^1 (3-2x)^3 dx$; (ii) $\int_0^{\frac{\pi}{4}} \tan^2\theta d\theta$.

Use the Tables to find, to three decimal places, the value of $\int_0^{\frac{\pi}{3}} \sin(2-x)dx$.

[42 marks.]

9. Find the least area (to the nearest square foot) of canvas that can be used to construct a conical tent whose capacity is 1,000 cubic feet. [42 marks.]

10. Trace the curve

$$y=(x-1)^3(x-2)-1,$$

giving particular attention to the part lying between the lines $x=1$, and $x=2$.

Mention where points of inflexion occur, and where y reaches its minimum value. [42 marks.]