AN ROINN OIDEACHAIS

(Department of Education.)

BRAINNSE AN MHEADHON-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1940.

HONOURS.

MATHEMATICS

(GEOMETRY)

FRIDAY, 14*th JUNE.*—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. If a transversal cuts the sides BC, CA, AB of a triangle at D, E, F respectively, prove that

$$\frac{BD}{DC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FB}=-1.$$

If two of the straight lines BE, CF, AD are the internal bisectors of two of the angles of the triangle, prove that the other straight line is the external bisector of the third angle.

[40 marks.]

2. A straight line AB is divided harmonically at C, D; prove that CD is divided harmonically at A, B.

If O is the mid-point of AB, prove that $OB^2=OC.OD$.

[40 marks.]

3. Show how to construct an equilateral triangle equal in area to a given square. Give proof.

[40 marks.]

4. (*a*) Find the general solutions of the equation $12\cos^3 x + 2\sin 2x = 11\cos x$.

(*b*) Show that the formula $n\pi + (-1)^n \frac{\pi}{6}$ represents the same series of angles as the formula $2n\pi + \frac{\pi}{2} \pm \frac{\pi}{3}$, where n takes all integral values (positive, negative and zero).

[40 marks.]

5. (*a*) Prove that

$$\tan^{-1}\frac{1}{3} + \frac{1}{2}\cos^{-1}\frac{3}{5} = \frac{\pi}{4};$$

(*b*) If $\theta = \cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha}$, prove that $\sin\theta = \tan^2\frac{\alpha}{2}$.

[40 marks.]

6. The inscribed circle of a triangle ABC touches the sides at D, E, F. The internal points of contact of the escribed circles with the sides are D′, E′, F′. Find, in terms of s, a, b, c, the segments into which D, E, F and D′, E′, F′ divide the sides. Hence, or otherwise, prove that the triangles DEF, D′E′F′ are equal in area.

[42 marks.]

7. Show that (i) $x - 3y = a$ represents a system of parallel straight lines for all values of a, and (ii) $bx + 2y = 7$ represents a system of straight lines passing through a fixed point for all values of b.

What is the equation of the line which is included in both systems ?

[42 marks.]

8. Prove that the length of the perpendicular from (x', y') to the straight line $x\cos\alpha + y\sin\alpha - p = 0$ is $x'\cos\alpha + y'\sin\alpha - p$.

Find the condition that the straight line $x\cos\alpha + y\sin\alpha - p = 0$ should touch the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

[42 marks.]

9. Prove that $3x^2+8xy-3y^2-7x+9y-6=0$ represents two straight lines at right angles and find the equation of the circumcircle of the quadrilateral formed by these two lines with the axes.

[42 marks.]

10. Prove that the straight line $y=mx+\frac{a}{m}$ touches the parabola $y^2=4ax$, whatever the value of m may be.

Hence, or otherwise, prove that the point of intersection of two tangents to a parabola which are at right angles to one another lies on the directrix.

[42 marks.]