AN ROINN OIDEACHAIS

(Department of Education.)

BRAINNSE AN MHEADHON-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1940.

HONOURS.

MATHEMATICS

(Algebra).

TUESDAY, 18*th JUNE*.—AFTERNOON, 3 TO 5.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the equations

(*a*) $x^2 - xy + 3y^2 = 15$,
$x^2 + xy - y^2 = 5$.

(*b*) $\sqrt[3]{5x-2} - \sqrt[3]{x+6} = \sqrt[3]{x-2}$.

[40 marks.]

2. Factorise

(i) $(b-c)^3(a-x)^3+(c-a)^3(b-x)^3+(a-b)^3(c-x)^3$;

(ii) $a^3(b-c)+b^3(c-a)+c^3(a-b)$.

[40 marks.]

3. Write down the first four terms in the expansion of $\left(1-\frac{1}{50}\right)^{\frac{1}{2}}$ and hence find the value of $\sqrt{2}$, correct to 6 decimal places. Show that 4 terms are sufficient for the purpose.

[40 marks.]

4. Find the sum of n terms of

(i) $1+3x+5x^2+7x^3+\ldots$.

(ii) $2.5+5.8+8.11+11.14+\ldots$.

[40 marks.]

5. In how many different ways can the letters of the word *infinitesimal* be arranged? In how many of these arrangements will the four *i*'s come together? In how many will no two of the *i*'s come together?

[40 marks.]

6. Differentiate—

(i) $(1+3x^2)^2$; (ii) $x\sqrt{1+3x^2}$; (iii) $x\sin x\cos x$.

Show that the differential coefficient of $\sin nx\sin^n x$ with respect to x is equal to $n\sin^{n-1}x\sin(n+1)x$.

[42 marks.]

7. Find the altitude of the cone of maximum volume which can be inscribed in a sphere of radius r.

[42 marks.]

8. Find the value of

(i) $\int_0^1 x^2(1+3x)dx$; (ii) $\int_0^1 \frac{x^2dx}{(1+x^3)^2}$;

(iii) $\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$; (iv) $\int_0^{\frac{\pi}{4}} \sin^2\theta\cos^2\theta d\theta$.

[42 marks.]

9. Find the area enclosed by the parabolas $y^2=kx$ and $x^2=ky$. Find also the volume generated by the revolution of this area about one of the axes.

[42 marks.]

10. Find the limits to the value of the function $\frac{(2x-1)(2x-3)}{5x^2+1}$ when x is real.

Draw a rough graph of the function, paying special attention to the maximum and minimum points and to the infinite branches.

[42 marks.]