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(Department of Education.)

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LEAVING CERTIFICATE EXAMINATION, 1939.

HONOURS.
MATHEMATICS
(GEOMETRY)

THURSDAY, 15*th JUNE*.—Morning, 10 a.m. to 12.30 p.m.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. A circle cuts a circle S_1 at A, B and a circle S_2 at C, D ; prove that AB, CD intersect on the radical axis of S_1 and S_2.

Hence, or otherwise, show how to construct the radical axis of two non-intersecting circles.

[40 marks.]

2. Explain the terms (i) pole and polar, (ii) conjugate points, (iii) self-conjugate triangle.

If a triangle is self-conjugate with respect to a circle, prove that its orthocentre is the centre of the circle and that the triangle is obtuse-angled.

[40 marks.]

3. P, Q, R are three points on a circle centre O. The diameter AOB bisects QR and intersects PQ, PR at M. and N. Prove that the triangles QOM, QON are similar and that N, M are harmonic conjugates of A, B.

[40 marks.]

4. Prove that

(i) $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$;

(ii) $\cos^{-1}\sqrt{\frac{2}{3}}+\cos^{-1}\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}=\frac{\pi}{3}$.

[40 marks.]

5. In a triangle ABC the line joining A to I, the centre of the inscribed circle, meets the circumcircle at P: prove that AI.IP=2Rr.

Prove also that

$$\frac{ab-r_1r_2}{r_3}=\frac{bc-r_2r_3}{r_1}=\frac{ca-r_3r_1}{r_2}.$$

[40 marks.]

6. Find the values of x in the range 0 to π for which the expression $x+\cos 2x$ has maximum or minimum values and draw a rough sketch of the curve $y=x+\cos 2x$ in that range.

[42 marks.]

7. The point P is on the line $3y=2x+1$ and the point Q is on the line $y=4x-3$. The middle point of PQ is $(0, -1)$. Find the equation of PQ.

[42 marks.]

8. Find the area of the triangle formed by the straight lines $2x^2-3xy-2y^2=0$ and $2x+5y=8$.

Find also the equation of the circle which passes through their points of intersection.

[42 marks.]

9. Find the condition that the straight line $y=mx+c$ may be a tangent to the circle $x^2+y^2=a^2$.

Find the equations of the tangents to the circle $x^2+y^2=13$ which are parallel to the straight line $2x-3y+4=0$.

[42 marks.]

10. Find the equation of the parabola whose focus is the point $(0, 0)$ and directrix the straight line $2x-y+1=0$.

Find the coordinates of the extremities of the latus rectum and draw a rough sketch of the curve.

[42 marks.]