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(Department of Education.)

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(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1939.

HONOURS.

MATHEMATICS

(Algebra).

MONDAY, 19th JUNE.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. (i) Solve the equation

$$\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}.$$

- (ii) Find the real solutions of

$$(x+y)^2 = 9xy - 4,$$

$$9xy(x+y) = 16.$$

[40 marks.]

2. Express $\sqrt{8}$ in the forms $2(1-a)^{-\frac{1}{2}}$ and $(1-b)^{-\frac{3}{2}}$ and hence show that

$$\sqrt{8} = 2 + \frac{1}{2} + \frac{1.3}{2.2^3} + \frac{1.3.5}{3.2^5} + \dots \text{ad inf.}$$

$$= 1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots \text{ad inf.}$$

[40 marks.]

3. Find the sum of (i) $2m-1$, (ii) $2m$ terms of the series

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots$$

where m is a positive integer.

Write down the sum of n terms of this series (a) when n is odd, (b) when n is even.

Hence find an expression for the sum of n terms for all positive integral values of n .

[40 marks.]

4. If $a+b+c+d=0$,
 prove that $a^3+b^3+c^3+d^3=3(bcd+cda+dab+abc)$,
 and that $(bc-ad)(ca-bd)(ab-cd)=(bcd+cda+dab+abc)^2$.

[40 marks.]

5. In a football match Kerry scored 2 goals and 6 points, and Galway scored 3 goals and 3 points. In how many different orders might (i) Kerry, (ii) Galway have compiled their respective scores and (iii) in how many different orders might all the scores have been made? [Points are scored singly in Gaelic football.]

[40 marks.]

6. Find, correct to two places of decimals, the negative root of the equation

$$x^3 - 2x + 7 = 0.$$

[42 marks.]

7. Differentiate

$$(i) \frac{1+x}{1-x}; \quad (ii) \sqrt{\frac{1+x}{1-x}}; \quad (iii) \sin^2 x \cos ax.$$

Find the limits to the value of x for which $3x^4 - 2x^3$ increases when x increases.

[42 marks.]

8. Show that $\int_a^b f(x)dx$ represents the area between the curve $y=f(x)$, the x -axis, and the ordinates $x=a$ and $x=b$.

Find the area between the curve $xy^2=(a-x)^2$, the x -axis, and the ordinates $x=0$ and $x=a$.

[42 marks.]

9. If $y=a+bx$, write $\int_0^1 \frac{dx}{(a+bx)^2}$ in terms of y , and in that way determine its value.

(a) Evaluate

$$(i) \int_0^1 x(3-2x)^{\frac{3}{2}} dx; \quad (ii) \int_0^{\frac{\pi}{2}} \sin^3 \cos x dx.$$

[42 marks.]

10. A conical glass containing water has its axis vertical and vertex downwards. If, through a small hole in the vertex, the water is allowed to escape at a uniform rate, prove that the rate at which the surface of the water in the glass descends is inversely proportional to the square of the height of the surface above the vertex.

[42 marks.]