AN ROINN OIDEACHAIS

(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1938.

HONOURS.

MATHEMATICS

(GEOMETRY)

FRIDAY, 17*th* *JUNE*.—Morning, 10 A.M. to 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. If a transversal cuts the sides AB, BC, AC of a triangle ABC in P, Q, R respectively, prove that

$$\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RA} = -1.$$

Prove also that if a transversal cuts the sides AB, BC, CD, DA of a quadrilateral ABCD in P, Q, R, S respectively, then

$$\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA} = 1.$$

[40 marks.]

2. Prove that the angle at which two curves cut is equal to the angle at which their inverses cut at the corresponding point of intersection.

What is the inverse of (i) two parallel straight lines, (ii) two perpendicular straight lines (with respect to a centre of inversion not on any of the given lines)?

[40 marks.]

3. Solve the equations

(i) $\sin^{-1}\frac{3}{x}+\sin^{-1}\frac{4}{5}=\sin^{-1}1$;

(ii) $\tan^{-1}(x+1)+\cot^{-1}(x-1)=\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{3}{5}$.

[40 marks.]

4. In a triangle ABC prove that

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

If the radii of the four circles that touch the sides of a triangle are in proportion, prove that the triangle is right-angled.

[40 marks.]

5. In a triangle the smallest angle is 45°, the tangents of the angles are in A.P., and the area is 5 square inches. Find the lengths of the sides.

[40 marks.]

6. The bisector of the angle A of a triangle ABC meets BC at K and the perpendicular to AK at K meets AB at H. Prove that AH is the harmonic mean between AB and AC.

Hence, or otherwise, prove that

$$\frac{1}{p}\cos\frac{A}{2}+\frac{1}{q}\cos\frac{B}{2}+\frac{1}{r}\cos\frac{C}{2}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c},$$

where p, q, r are the bisectors of the angles A, B, C respectively.

[42 marks.]

7. Prove that the perpendicular distance of the point (x', y') from the straight line $ax+by+c=0$ is $\pm\dfrac{ax'+by'+c}{\sqrt{a^2+b^2}}$.

Find the equations of the two bisectors of the angles between the straight lines $3x-4y+1=0$ and $5x+12y-1=0$ and show that they are at right angles.

[42 marks.]

8. Show that the quadrilateral whose vertices are (0, 1); (1, 3); $(4\frac{1}{2}, 2\frac{1}{2})$; (1, −1) respectively is cyclic and find its area.

[42 marks.]

9. Show that $x^2+y^2+2\lambda x+9=0$ represents a system of coaxal circles, where λ may have any value, and find the coordinates of the limiting points of the system.

Find the equation of the corresponding orthogonal system of coaxal circles.

[42 marks.]

10. Write down or find the equation of the tangent at the point (x', y') on the parabola $y^2=4ax$. Hence find the equation of the normal at that point.

P is any point on a parabola and the normal at P meets the axis at G: prove that the locus of the middle point of PG is another parabola.

[42 marks.]