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(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1938.

HONOURS.
MATHEMATICS
(Algebra).

TUESDAY, 21*st* *JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. (i) Solve the equations

$$x+y+\sqrt{x^2-y^2}=1,$$
$$y\sqrt{x^2-y^2}=3.$$

(ii) If the equations

$$ax+by=1,$$
$$cx^2+dy^2=1$$

have only one solution, prove that the solution is $x=\frac{a}{c}$ and $y=\frac{b}{d}$.

[40 marks]

2. (i) Factorise $(a+b-c)^3-a^3-b^3+c^3$.

(ii) Find the four linear factors of $a^4+b^4+c^4-2b^2c^2-2c^2a^2-2a^2b^2$.

[40 marks.]

3. The nth term of the series 1, -3, -5, ... is of the form an^2+bn+c; find the sum of n terms.

Find the terms which are equal and show that no terms are equal after the sixth term.

[40 marks.]

4. If x is so small that its square and higher powers may be neglected, express in the form $l+mx$ the value of

$$\frac{(1+3x)^{\frac{1}{3}}\sqrt{4-5x}}{(3+2x)\sqrt{1+3x}}.$$

[40 marks.]

5. How many numbers less than 1000 can be formed in which no digit exceeds 6 ?

In how many of these will the digits be all different ?

[40 marks.]

6. A and B, starting at the same time, walk at uniform rates, the former from X to Y and the latter from Y to X. Prove that their times for the whole distance are in the ratio of the square roots of the times taken by them to complete their respective journeys after they have met.

If A and B travel at speeds in the ratio of 4 : 3 and meet 24 minutes after starting, find their times for the whole journey.

[42 marks.]

7. Find from first principles the differential coefficient of $\dfrac{7x+2}{3x-4}$.

Differentiate (i) $\sqrt{\dfrac{7x+2}{3x-4}}$,

(ii) $x\tan x+2x\tan 2x$.

[42 marks.]

8. Find the value of $\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$.

Show by putting $x=a\cos\theta$ that

$$\int_0^a \sqrt{a^2-x^2}\, dx = \int_0^{\frac{\pi}{2}} a^2\sin^2\theta d\theta$$

and hence deduce that the area of the circle $x^2+y^2=a^2$ is πa^2.

[42 marks.]

9. Trace the curve $y=x^2-x^4$.

Find the area between the x-axis and the part of the curve above it.

[42 marks.]

10. Find the volume of the greatest right cone which can be cut from a solid sphere of radius r.

[42 marks.]