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LEAVING CERTIFICATE EXAMINATION, 1937.

HONOURS.

MATHEMATICS
(GEOMETRY)

THURSDAY, 17*th* *JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Explain what is meant by (i) radical axis of two circles; (ii) coaxal circles; (iii) limiting points.

Given two circles of a system of non-intersecting coaxal circles, show how the radical axis and the limiting points can be found.

[40 marks.]

2. If a straight line AB is divided internally at X and externally at Y in the same ratio, prove that AB is the harmonic mean between AX and AY.

If M is the middle point of AB, prove that $MB^2 = MX.MY$.

[40 marks.]

3. Show how to find a point P in the base AB of a triangle ABC so that $AP : PB = AC^2 : CB^2$.

By means of the identity $(ax+by)^2+(bx-ay)^2=(a^2+b^2)(x^2+y^2)$, or otherwise, prove that P is the point in AB such that the sum of the squares on the perpendiculars drawn from it to the sides AC, BC is a minimum.

[40 marks.]

4. (i) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that
$$x^2+y^2+z^2+2xyz=1.$$

(ii) Prove that
$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}.$$

[40 marks.]

5. In a triangle prove that

(i) $r=(s-a)\tan\frac{1}{2}A$;

(ii) $r_1=s\tan\frac{1}{2}A$;

(iii) $r=4R\sin\frac{1}{2}A\sin\frac{1}{2}B\sin\frac{1}{2}C$.

[40 marks.]

6. Solve generally the equations :

(i) $\sin\theta\sec 2\theta+\cos\theta=\cos 3\theta$;

(ii) $3\cos\theta-2\sin\theta=1$.

[42 marks.]

7. The coordinates of the vertices of a triangle are $(1, 2)$; $(-3, 1)$; $(2, -3)$. Find the area of the triangle and the size of the greatest angle.

[42 marks.]

8. Prove that the straight lines joining the origin to the points of intersection of the straight line $4x+3y=10$ and the circle $x^2+y^2+3x+6y-20=0$ are at right angles.

[42 marks.]

9. Prove that $y=mx\pm a\sqrt{1+m^2}$ is a tangent to the circle $x^2+y^2=a^2$ for all values of m.

Two tangents at right angles to one another are drawn to the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$ respectively ; find the locus of their point of intersection.

[42 marks.]

10. Find the equation of the axis and the coordinates of the vertex of the parabola $(x-2y)^2+4x-3y=0$.

Find also the equation of the tangent at the origin and draw a rough sketch of the curve.

[42 marks.]