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(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1937.

HONOURS.

MATHEMATICS
(Algebra).

MONDAY, 21*st JUNE.*—Afternoon, 3.30 to 6 p.m.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. (i) Factorise $2x^2 - 21xy - 11y^2 - x + 34y - 3$;

   (ii) Solve the equations :
   $$2x^2 - 21xy - 11y^2 - x + 34y - 3 = 0,$$
   $$2x^2 - xy - y^2 = 38.$$
   [40 marks.]

2. (i) Prove that b^2 is greater than, equal to, or less than ac when a, b, c are in A.P., G.P., and H.P. respectively.

   (ii) Find the sum to infinity of $\frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \ldots$
   [40 marks.]

3. Prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n}{6}(n+1)(2n+1)$.

   The first term of a series is 1 and the $(n+1)$th term exceeds the nth term by $4n+1$; find the nth term and the sum of n terms.
   [40 marks.]

4. (a) In how many ways can a committee of 5 be chosen from 6 ladies and 8 gentlemen if the committee is to contain at least 2 ladies ?

   (b) In how many ways can 5 books be arranged in a row so that a particular book A may (i) be on the immediate left of another book B, (ii) be on the left of B but not necessarily on the immediate left of B ?
   [40 marks.]

5. Prove that the coefficient of the nth term in the expansion of $(1-x)^{-n}$ is twice the coefficient of the $(n-1)$th term.

Use the Binomial Theorem to find the sum to infinity of

$$1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+\ \cdots$$

[40 marks.]

6. If $y=a(\theta+\sin\theta)$

and $\quad x=a(1-\cos\theta)$,

prove that $\dfrac{dy}{dx}=\pm\ \dfrac{\sqrt{2ax-x^2}}{x}$.

[42 marks.]

7. Prove that the volume of a cone of height h and radius of base a is $\frac{1}{3}\pi a^2 h$.

[42 marks.]

8. Find the angle which the tangent to the curve $y=x(x+2)(x-1)^2$ at the origin makes with the axis of x.

Find where the tangent meets the curve again, find the maximum and minimum points, and draw a rough sketch of the curve.

[42 marks].

9. Find from first principles the derivative of $\sin 2x$.

Differentiate (i) $x\sin 2x$; (ii) $x\sin 2x\cos^2 x$; (iii)$x^2\div\sqrt{1+x^2}$.

[42 marks.]

10. Find the values of :

(i) $\displaystyle\int_{-1}^{+1}(x+1)^3 dx$;   (ii) $\displaystyle\int_{0}^{1}\sqrt{2x+1}\ \ dx$;

(iii) $\displaystyle\int_{0}^{\frac{\pi}{2}}\sin 3\theta d\theta$; $\displaystyle\int_{0}^{\frac{\pi}{2}}\sin^3\theta d\theta$.

[42 marks].