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(Department of Education).

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LEAVING CERTIFICATE EXAMINATION, 1936.

HONOURS.

MATHEMATICS
(GEOMETRY)

THURSDAY, 18*th JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. What is meant by "pole and polar"?

If A and B are two points and if the polar of A with respect to a circle passes through B, prove that the polar of B passes through A.

[40 marks.]

2. Prove *either* Ceva's Theorem *or* Menelaus' Theorem.

O is a point inside a triangle ABC and AO, BO, CO produced meet the sides in D, E, F respectively. If BD=2DC and CE=2AE, prove that BF=4FA and find the ratio of AO to OD.

[40 marks.]

3. Prove that the areas of similar triangles are in proportion to the squares on their corresponding sides.

From a point P on the outer of two concentric circles tangents PA, PB are drawn to the inner. PA, BA meet the outer again in D, C respectively. Prove that $CA : CB = CD^2 : CP^2$.

[40 marks.]

4. Prove (i) $\sec^{-1} 3 = 2 \cot^{-1}\sqrt{2}$,

$$\text{(ii)}\ \tan^{-1}\frac{a-b}{ab+1} + \tan^{-1}\frac{b-c}{bc+1} + \tan^{-1}\frac{c-a}{ca+1} = 0$$

[40 marks.]

5. Solve generally the equation

$$2\cos\theta+\frac{1}{2\cos\theta}=2\tfrac{1}{2}.$$

Find the values of θ that make

$$2\cos\theta+\frac{1}{2\cos\theta}\text{ a minimum.}$$

[40 marks.]

6. I_1, I_2, I_3 are the centres of the escribed circles of the triangle ABC. Prove (i) that $\frac{\pi}{2}-\frac{A}{2}$, $\frac{\pi}{2}-\frac{B}{2}$, $\frac{\pi}{2}-\frac{C}{2}$ are the angles and (ii) that $a\operatorname{cosec}\frac{A}{2}$, $b\operatorname{cosec}\frac{B}{2}$, $c\operatorname{cosec}\frac{C}{2}$ are the sides of the triangle $I_1 I_2 I_3$.

[42 marks.]

7. Through the point (3, 4) two straight lines are drawn each making the angle 45° with the straight line $x-2y=1$. Find the equations of these lines.

Find also the area of the triangle formed by the three straight lines.

[42 marks.]

8. Prove that the equation of the tangent at the point (x^1, y^1) of the circle $x^2+y^2=c^2$ is $xx^1+yy^1=c^2$.

If a, b are the intercepts made on the axes by this tangent, prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$.

[42 marks.]

9. P is a point such that the sum of the squares on the straight lines joining P to the vertices of a given triangle is constant. Prove that the locus of P is a circle whose centre is at the centroid of the triangle.

Deduce that the least value of the sum of the squares on the straight lines joining any point to the vertices of a triangle ABC is $\frac{1}{3}(a^2+b^2+c^2)$.

[42 marks.]

10. Find the equation of the locus of the centre of a circle which passes through the point (1, —3) and touches the straight line $y-2x+1=0$.

Prove that the locus is a parabola and find the equation of its axis and the length of its latus rectum.

[42 marks.]