ÁN ROINN OIDEACHAIS
(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1935.

HONOURS.

MATHEMATICS
(GEOMETRY)

FRIDAY, 14th JUNE.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. ABC is a triangle in which A is an obtuse-angle. Show how to draw a straight line AD from A to BC so that (i) $AD^2 = BD.DC$; (ii) $AD^2 = 2BD.DC$.

[40 marks.]

2. Prove that the inverse of a straight line is a circle through the centre of inversion and that the line is the radical axis of its inverse and the circle of inversion.

[40 marks.]

3. If the points of section of a pencil of four rays by a transversal form a harmonic range, prove that the points of section by each other transversal form a harmonic range.

[40 marks.]

4. Prove that

$$5 \tan^{-1} \frac{1}{7} + 2\tan^{-1}\frac{3}{79} = 3 \tan^{-1} \frac{1}{7} + 2\tan^{-1}\frac{2}{11} = \frac{\pi}{4}.$$

[40 marks.]

5. Find the maximum and minimum values of

(i) $a\cos\theta + b\sin\theta$;

(ii) $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$.

[40 marks.]

6. In a triangle prove that

$$\text{(i)}\ \frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3};$$

$$\text{(ii)}\ 4R=r_1+r_2+r_3-r.$$

[42 marks.]

7. Find the angle between the two straight lines given by

$$3x^2-20\,xy+12y^2+22x+12y-45=0.$$

To which line is (5, 8) the nearer?

[42 marks.]

8. Find the coordinates of the middle point of the straight line joining (x_1, y_1) and (x_2, y_2).

Three of the vertices of a parallelogram are (0, 1); (−2, 3); (−3, −1). Find the coordinates of the fourth vertex in all its possible positions. Find also the area of the parallelogram.

[42 marks.]

9. Show that the tangents from the point (6, −1) to the circle $x^2+y^2-4x+2y+1=0$ are inclined at an angle of 60°, and find the length of the chord of contact.

[42 marks.]

10. TA, TB are tangents to a circle and any secant TCD meets the circle at C, D. Q is the middle point of CD. Prove that TQ bisects the angle AQB and that TQ $\propto$ AQ+BQ.

[42 marks.]