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(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
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LEAVING CERTIFICATE EXAMINATION, 1935.

HONOURS.

MATHEMATICS
(Algebra).

TUESDAY, 18th JUNE.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations

(i) $5xy = 6(x+y)$,
$2yz = 15(y+z)$,
$2zx = 10(z+x)$.

(ii) $x^{\frac{1}{5}} + y^{\frac{1}{6}} = 5$,
$x^{\frac{3}{5}} + y^{\frac{1}{2}} = 35$.

[40 marks.]

2. Prove the Binomial Theorem for a positive integral exponent. Prove that $\frac{62}{61}$ is a better approximation to the value of $\sqrt[3]{\frac{147}{140}}$ than $\frac{61}{60}$.

[40 marks.]

3. Find the sum of n terms of the series $1-3x+5x^2-7x^3+\ldots$

Prove that the series whose nth term is $\frac{n^2}{2^n}$ is convergent.

[40 marks.]

4. Twelve papers are to be given in an examination, three of them in Mathematics. In how many different orders may the papers be given provided that, (i) the three Mathematical papers may not all come consecutively; (ii) no two Mathematical papers may come consecutively.

[40 marks.]

5. Trace the curves

(i) $y = x(2-x)^2$;
(ii) $y = x^2(2-x)^2$.

Special attention should be given to maximum and minimum points and to points of inflexion.

[42 marks.]

6. The sides of a triangle are in A. P. and its area is to that of an equilateral triangle of the same perimeter as 3 is to 5. Prove that the greatest angle in the triangle is 120°.

[42 marks.]

7. Find from first principles the differential coefficient of $\sqrt{x}$.

Differentiate (i) $\sqrt{2x^2-1}$; (ii) $x \sin^2 2x$.

[42 marks.]

8. Explain geometrically why the value of $\int_{-1}^{1} (2+mx)dx$ is independent of m.

Find the values of

$$\text{(i)} \int_1^3 \left(x^2 - \frac{1}{x^2}\right) dx; \quad \text{(ii)} \int_0^{\frac{\pi}{2}} \sin^3 x \, dx.$$

[42 marks.]

9. AB, BC are two roads at right angles. A person proceeding from A to any point in BC can go by road or directly across country at rates proportional to 7 : 5 respectively. Prove that there are two solutions to the problem of finding a point in BC which he can reach in the same time either by road or across country; and that, if P, Q are the points resulting therefrom, for any point in BC between P and Q the quicker route is across country and that for all other points in BC (except P, Q) the quicker route is by road.

[42 marks.]