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LEAVING CERTIFICATE EXAMINATION, 1933.

HONOURS.

MATHEMATICS
(Algebra).

TUESDAY, 20th JUNE.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. All questions carry equal marks. Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations :

   (i) $\sqrt[3]{x} - \sqrt[3]{y} = 2$

   $x - y = 98.$

   (ii) $\sqrt{2x^2 + 3x + 5} + 2x^2 + 3x - 1 = 0.$

2. When is a function in two variables said to be (i) symmetrical, (ii) homogeneous ? Give examples. Write down a homogeneous symmetrical function of the second degree in three variables.

   Factorize $(a-b)(a+b)^4 + (b-c)(b+c)^4 + (c-a)(c+a)^4.$

3. Sum to n terms the series :

   (i) $x^2 + (x+1)^2 + (x+2)^2 + \ \cdots$

   (ii) $1.2.4 + 2.3.5 + 3.4.6 + \ \cdots$

4. In how many ways can a party of eight be chosen from 6 men and 6 women ? In how many of these will there be an equal number of men and women ? In how many will there be at least one woman to each man ?

5. Find, graphically or otherwise, approximations to the roots of $x^3 - 5x + 3 = 0$, giving one root correct to two decimal places and the others to the nearest integer.

6. What is meant by Convergency of a series ? State and prove any test for convergency.

   For what values of x is the series

   $$\frac{x}{2x-1} + \left(\frac{x}{2x-1}\right)^2 + \left(\frac{x}{2x-1}\right)^3 + \cdots$$

convergent ?

7. Find the first three terms in the expansion of $\dfrac{\sqrt[3]{1+x}}{1-x}$ in ascending powers of x. Hence find the value of the expression to three decimal places when $x = \cdot 02$.

8. Prove that $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{1}{v^2}\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right).$

Differentiate   (i) $\dfrac{x^3}{x^2+1}$,   (ii) $\dfrac{1}{\sin x \cos x}$.

9. Evaluate and interpret geometrically $\displaystyle\int_0^{\pi} \sin x\, dx.$

Find the area enclosed by the curve $y=(x-1)(2x-5)$ and the axis of x.

10. A spherical hailstone melts at a rate proportional to the area of its surface. Prove that the diameter diminishes uniformly with the time. If the total time taken to melt a hailstone is t seconds, find in terms of t, to 2 decimal places, the time taken to melt half of it.