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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1932.

HONOURS.

MATHEMATICS (II).

THURSDAY, 2nd JUNE.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. PQ meets the sides AB, AC of an isosceles triangle ABC in P, Q and is parallel to the base BC; prove that $BQ^2 - QC^2 = PQ \cdot BC$.

Prove that the converse theorem (*i.e.* given $BQ^2 - QC^2 = PQ \cdot BC$, to prove PQ parallel to BC) may not be true.

2. An equiangular polygon of n sides is inscribed in a circle. Prove that alternate sides are equal and that the polygon is not necessarily equilateral except when n is odd.

3. Prove that $\tan \frac{45^\circ + \theta}{2} \tan \frac{45^\circ - \theta}{2} = \frac{\sqrt{2} \cos \theta - 1}{\sqrt{2} \cos \theta + 1}$ and hence prove that $\tan 7\frac{1}{2}^\circ \tan 37\frac{1}{2}^\circ = (\sqrt{3} - \sqrt{2})^2$.

4. Prove that $\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$.

If the side of a regular pentagon is a inches, find in terms of a , without using the Tables, the radii of the inscribed and circumscribed circles.

5. Find from first principles the differential coefficient of $\sin x$. Differentiate (i) $x \sin x$, (ii) $(x \sin x)^n$.

6. Evaluate $\int x^2 dx$.

Find the area bounded by the curve $y=x^2$, the x -axis and the ordinates $x=1$, $x=3$. Find also the volume generated by the revolution of that area about the x -axis.

7. A tin can is made in the form of a cylinder. Find the relation between the diameter of the base and the height when the volume is constant and the least amount of tin is used.

8. Prove that $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$ or $\pi + \tan^{-1} \frac{a+b}{1-ab}$

Illustrate graphically the change in sign and magnitude of the function $3\sin x + 4\cos x$.

9. Use the equation $\tan 2\theta \tan 3\theta = 1$ to show that $\tan 18^\circ$ is a root of the equation

$$5x^4 - 10x^2 + 1 = 0$$

and hence or otherwise find all the roots to four decimal places.