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LEAVING CERTIFICATE EXAMINATION, 1931.

HONOURS.

MATHEMATICS (II).

MONDAY, 15*th JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Prove the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and use it to find, to at least 3 decimal places, the three roots of the equation $20x^3 - 15x - 1 = 0$.

2. Find one value of x which satisfies the equation $2\tan^{-1} 2x - \cos^{-1} x = \sin^{-1} x$.

Solve generally the equation $a\cos\theta + b\sin\theta = c$, and find the maximum and minimum values of $a\cos\theta + b\sin\theta$.

3. Prove that in a triangle (using the usual notation) $\frac{a}{h} = \frac{2\sin A}{\cos A + \cos(B-C)}$, where a is the base and h the height.

Find the angles of a triangle, given $a=5$, $h=3$, $B-C=10°$.

4. Show how the first derivative of a function of one variable leads to the determination of its maximum or minimum values, if any.

Part of a ship's expenses on a certain voyage are estimated to vary as the time taken and the remainder as the average velocity. When the voyage takes 15 days the cost is £1,098 : when it takes 27 days the cost is £1,170. Find the least cost of the voyage and the corresponding time taken.

5. Find, by *integration*, the area of a circle of radius r.

A vessel is of such dimensions that when it is filled with water to a depth of x inches the area of the surface of the water is $3+2x+\frac{x^2}{3}$ sq. inches. Find the number of cubic inches of water in the vessel when it is filled to a depth of 6 inches.

6. Draw rough sketches of the curves

(i) $y=8x^3-24x+1$

(ii) $y=(1+x)^2(4-x^2)$

giving special attention to their maximum and minimum points and points of inflexion.

7. Show how geometrical constructions can be found for (i) $\sqrt{a}$, (ii) $\sqrt{a+\sqrt{b}}$, where a, b are positive rational numbers. Show that these methods will lead to geometrical constructions for $\sqrt[n]{a}$ when n is any integral power of 2.

8. Use the approximation $\sin\theta=\theta-\frac{\theta^3}{\underline{3}}$ (where θ is measured in radians) to show that the length of an arc of a circle, subtending angle θ at the centre, is equal to $\frac{1}{3}(8b-a)$ approximately, where a is the length of the chord of the arc and b the length of the chord of half the arc.

Show that in a circle whose radius is 1 mile the error from above formula is only about 6 feet for an angle of 90°.

9. Find the locus of a point which moves so that its distances from two given points are in a given ratio.

Show how to construct an equilateral triangle that will be such that the distances of its vertices from a fixed point are given lengths.