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HONOURS.

MATHEMATICS (I).

THURSDAY, 11*th JUNE.*—10 A.M. TO 12.30 P.M.

Six questions may be answered.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations :

(i) $a(b-x)^3+b(x-a)^3+x(a-b)^3=0$.

(ii) $(x+2)(x+3)(2x-5)(2x-7)=126$.

2. Show that in a quadratic equation with real coefficients both roots are imaginary, if one is.

Prove also that in the equation $x^2+(a+bi)x+c+di=0$, where a, b, c, d are real and not zero and $i\equiv\sqrt{-1}$, both roots can never be real and find the condition that one can.

3. The diagonals of a regular pentagon A_1 intersect so as to form another regular pentagon A_2, the diagonals of A_2 intersect so as to form another regular pentagon A_3, and so on. Show that the areas of $A_1, A_2, A_3 \ldots$ are in G.P., and find the least number of such pentagons which should be formed so that the area of the last will be less than one-millionth the area of the first.

4. Find the sum of n terms of the series whose nth terms are (i) n^2, (ii) n^2-n+1.

Find also the sum of all the products taken two at a time of the first n natural numbers.

5. Explain any method for the approximate determination of the real roots of an equation with numerical coefficients.

Find, to three decimal places, one root of the equation

$$x^4-3x+1=0.$$

6. Prove that the series $1.2+2.3.\frac{1}{2}+3.4.\frac{1}{2^2}+4.5.\frac{1}{2^3}+\ . \ . \ .$ is convergent, and use the Binomial Theorem to find its sum to infinity.

7. Show how the angle between two straight lines whose equations are given can be found.

Find (i) the angles, (ii) the area of the triangle formed by the intersection of the straight lines $x-2y-5=0$, $5x-2y-6=0$, $2x+y+10=0$.

8. Show that if $\alpha=0$, $\beta=0$, $\gamma=0$ are the equations of the sides of a triangle, p and q can be found so that $\beta\gamma+p\gamma\alpha+q\alpha\beta=0$ is the equation of the circumcircle.

Hence or otherwise find the equation of the circumcircle of the triangle formed by the straight lines $2x-y=0$, $2x+3y+3=0$, $3x-2y+3=0$.

9. What is a parabola ? Find the equation of the parabola whose focus is the point (h, k) and directrix the straight line $ax+by+c=0$. Find also the equation of the axis of the parabola and the length of its latus rectum.

10. Find the equations of the common tangents of the circles $x^2+y^2=16$ and $(x-10)^2+y^2=4$.