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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1929.

HONOURS

MATHEMATICS (II).

TUESDAY, 18*th* *JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. Question 6 (*a*) or 6 (*b*) may be answered, but not both. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Illustrate geometrically the meaning of the derivative of a function.

A quadratic function of x becomes equal to -5 when $x = 3$ and the tangents to its curve at $x = -1$ and $x = 2$ make angles $\tan^{-1} 7$ and $\tan^{-1} -5$ respectively with the positive direction of the x-axis. Find the value of the function at its turning point.

2. Find from *first principles* the derivatives of $\frac{1}{x^2+1}$ and of $\sec x$. If $y = \sec^{-1} \frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$ in its simplest form.

3. A conical tent is to have a given volume, V. Find what is the ratio of its height to its base radius when the least possible amount of canvas is used.

4. Make a rough sketch of the graph of the function $x^4 - a^2x^2 + a^2y^2 = 0$, and calculate the volume of the solid generated by rotating round the x-axis the area enclosed by the curve.

5. Find θ from the relation

$$\tan^{-1}\theta = \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{3},$$

and solve the equation

$$\tan^{-1}\frac{x}{a} + \tan^{-1}\frac{x}{b} + \tan^{-1}\frac{x}{c} = \frac{\pi}{2}.$$

6 (*a*). The angle in a segment cut off from a circle by a chord AB, of length a, is φ. A tangent PQ meets BA, produced, in P and the perpendicular through B to the chord AB in Q. If $\angle$ BPQ $= \theta$, find the simplest expression for PQ in terms of a, θ and φ.

Or

6 (*b*). With the usual notation for a triangle and its circles prove that the area of the triangle $I_1I_2I_3 = 8R^2 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$.

7. Show that the sum of the rectangles contained by the opposite sides of a cyclic quadrilateral is equal to the rectangle contained by the diagonals.

P is a point on the circumcircle of an equilateral triangle ABC: prove that one of the lines PA, PB, PC is equal to the sum of the other two.

8. D and E are points on the sides AB and AC respectively of the triangle ABC such that AD : DB $= m : n$, and AE : EC $= p : q$. BE and CD intersect at O. Prove that the triangle AOC : triangle BOC $= m : n$ and express the ratios BO : OE and CO : OD in terms of m, n, p, q.

9. O is a fixed point and OP_1P_2 is any secant through O intersecting a fixed circle in P_1 and P_2. Q is a point on P_1P_2 such that OQ is equal to (i) the arithmetic, (ii) the geometric, and (iii) the harmonic mean between OP_1 and OP_2; in each case determine geometrically or analytically the locus of Q.