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(Department of Education).

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(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1929.

HONOURS

MATHEMATICS (I).

THURSDAY, 13*th* *JUNE.*—10 A.M. TO 12.30 P.M.

Six questions may be answered.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. (*a*) If $(a+b)^2 = 4x^2ab$, $(b+c)^2 = 4y^2bc$, and $(c+a)^2=4z^2ca$, prove that $x^2+y^2+z^2 = 1 \pm 2xyz$.

(*b*) Factorise $6x^2 - 5xy - 6y^2 + 7x + 22y - 20$.

2. Solve the equations :

(i) $\sqrt{1-x+x^2} - \sqrt{1+x+x^2} = a$.

(ii)
$$x + y + z = 1$$
$$x^2 + y^2 + z^2 = 21$$
$$xyz = 8$$

3. (i) Find the sum of the squares of the first n natural numbers.

(ii) Find the nth term of a series whose initial terms are the same as 1, 2, 4, 7, 11, 16, 22 . . . and sum your series to n terms.

4. Find the term independent of x in the expansion of

$$(1 + x)^5 \times \left(1 + \frac{1}{x}\right)^5.$$

If $(1+x)^n = c_0+c_1 x+c_2 x^2+ \ . \ . \ . \ + c_n x^n$, prove that

$$c_0^2 + c_1^2 + c_2^2 + \ . \ . \ . \ + c_n^2 = \frac{(2n)!}{n!\, n!}$$

5. Calculate to *four* significant figures the real root of the equation $x^3 - 5x^2 + 9x - 8 = 0$.

6. Find the co-ordinates of the points dividing in a given ratio the line joining two given points.

A straight line is such that its intercept between the x and y axes is divided in the ratio $m : n$ at a given point (a, b); find the equation of the line.

7. Derive an expression for the length of the perpendicular from the point $(x_1\ y_1)$ to the line $Ax + By + C = 0$.

Determine, without drawing a figure, whether the points $(-8, 10)$ and $(0, 0)$ are on the same or on opposite sides of the line $13x - 16y + 208 = 0$. Similarly show whether the point $(-8, 10)$ lies within or without the circle

$$x^2 + y^2 + 10x - 14y + 58 = 0.$$

8. (x_1, y_1) and (x_2, y_2) are the extremities of the diameter of a circle. Obtain equations to the straight lines joining each of these points to the point (ξ, η) on the circle and show that the equation to the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

Find the equation of the circle having as diameter the chord intercepted on the line $x + 3y = 35$ by the circle $x^2 + y^2 = 125$.

9. Define a parabola and obtain its equation in the form $y^2 = 4ax$.

From a point P $(at^2, 2at)$ on the parabola $y^2 = 4ax$ a normal is drawn intersecting the x- axis in Q. Show that the mid-point of PQ describes a parabola as P moves on $y^2 = 4ax$.