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(Department of Education).

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(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1928.

HONOURS

MATHEMATICS (II).

MONDAY, 18*th* *JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. Question 3 (*a*) or 3 (*b*) may be answered, but not both. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = n\pi$.

If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \pi$, prove that $yz + zx + xy = 1$.

2. In a quadrilateral ABCD, $AB = 4 \cdot 28$ inches, $BC = 5 \cdot 37$ inches, and the angles ABC, BCD, CDA are 117, 92, 49 degrees respectively. Calculate the length of AD.

3 (*a*). Solve the equations :—

(i) $\sin\theta + \cos\theta = \cos 2\theta$.

(ii) $\tan^{-1} x + \tan^{-1} 2x = n\pi + \dfrac{3\pi}{4}$

(For full marks the general solutions should be given).

Or

3 (*b*). Find the distance between the centres of the inscribed and circumscribed circles of a triangle in terms of the radii of these circles.

4. Find *from first principles* the derivatives of (i) x^3, (ii) $\tan x$.

The inside of a wine-glass is in the shape of a right-circular cone of semi-vertical angle 45°. Wine is poured into it at the rate of 2 cubic inches per second. Find the rate at which the level is rising at the end of 3 seconds.

5. Find the maximum and minimum ordinates and the gradient at the point of inflexion on the curve $y=x(x-1)(x-2)$. Sketch the curve.

6. Prove by integration that the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius.

7. The curve $y=\mathrm{A}x-\mathrm{B}x^3$ is such that the area bounded by the curve, the x-axis, and the ordinates $x=1$, $x=2$ is 30 units. This is also true in the case of the ordinates $x=2$, $x=3$. Find A and B and the area enclosed by the x-axis, the curve above the x-axis and the ordinates $x=1$, $x=3$.

8. Prove that similar triangles are to one another as the squares on corresponding sides.

Construct an equilateral triangle equal in area to a square of side 2 inches.

9. If three points X, Y, Z lying respectively on the sides BC, CA, AB of a triangle ABC are collinear, prove that the ratio compounded of the ratios AZ : ZB, BX : XC, CY : YA is equal to unity.

Prove also that if two triangles are so placed that their vertices connect concurrently, their corresponding sides intersect collinearly.