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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1927.

HONOURS

MATHEMATICS (II).

TUESDAY, 21*st JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. 5 (*a*) *or* 5 (*b*) may be attempted, but not both. All questions carry equal marks.

Tables of Measures, Constants and Formulae and Logarithmic Tables may be obtained from the Superintendent.

1. If u and v are functions of x, find the derivative of $u\,v$ in terms of the derivatives of u and v.

If $(1 + x^2) \sin y = 2x$,

show that $\dfrac{dy}{dx} = \pm \dfrac{2}{1 + x^2}$.

2. Find the maximum and minimum values of the function $x^3 - 3x$. Draw a rough diagram of the curve $y = x^3 - 3x$.

3. If the curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $\dfrac{x^2}{c^2} + \dfrac{y^2}{d^2} = 1$ intersect at right angles, show that $a^2 - b^2 = c^2 - d^2$.

4. Find the area between the parallels $x = 0$ and $x = b$ bounded by the curve $y^2 = 4ax$ and the x-axis. If this area is made to revolve about the x-axis, find the volume generated.

5. (*a*) Give the Argand method of representing geometrically the quantity $a + i\,b$ where $i = \sqrt{-1}$. What represents the modulus of the quantity ?

What complex quantity would be represented by the centroid of a triangle having vertices $a + i\,b$, $a_1 + i\,b_1$ and $a_2 + i\,b_2$?

Or

(*b*) I_1, I_2, I_3 are the centres of the escribed circles of the triangle ABC. Show that I_1 BC, I_2 CA, I_3 AB are similar triangles whose areas are as $\sin^2\frac{A}{2} : \sin^2\frac{B}{2} : \sin^2\frac{C}{2}$.

6. $\text{Sin}^{-1} P + \sin^{-1} Q = \sin^{-1} R$,

where $P = \frac{2ab}{a^2+b^2}$ and $Q = \frac{2cd}{c^2+d^2}$.

Find the value of R in terms of a, b, c, d in the form

$$\frac{2AB}{A^2+B^2}.$$

Show that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{56}{65}$.

7. A and B are two points in a line with the centre O of a circle and such that OA.OB=square on the radius of the circle. Show that any circle passing through A and B will cut the given circle orthogonally.

Find the locus of the centre of a circle which cuts two given non-intersecting circles orthogonally.

8. P is the mid-point of the arc AB of a circle and D any other point on the circle. Show that PC. PD=PB² where C is the intersection of the lines AB and PD.

Given the base, vertical angle and the length of the bisector (from the vertex to the base) of the vertical angle of a triangle, show how to construct the triangle.