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LEAVING CERTIFICATE EXAMINATION, 1927.

HONOURS

MATHEMATICS (I).

FRIDAY, 17*th JUNE*.—10 A.M. TO 12.30 P.M.

Tables of Measures, Constants and Formulae, and Logarithmic Tables may be obtained from the Superintendent.

Six questions may be answered. 4(*a*) *or* 4(*b*) may be attempted, but not both.

All questions carry equal marks.

1. Show how to solve the equation

$$a\,(x^4+1)+b\,(x^3+x)+cx^2=0.$$

Complete the solution when $a=15$, $b=14$, $c=-90$.

2. Find the coefficient of x^7 in the expansion of $(1-4x)^{\frac{11}{4}}$ and the term independent of x in $\left(\frac{4}{3}x^2-\frac{3}{2x}\right)^9$.

3. Find to three places of decimals the real root of

$$x^3-x^2-1=0.$$

4. (*a*) Prove that a series of positive terms is convergent if, after any particular term, the ratio of each term to the preceding is always less than some fixed quantity which is itself less than unity.

Hence or otherwise test the convergency of the series:—

$$1+\frac{1}{4}+\frac{1\cdot4}{4\cdot8}+\frac{1\cdot4\cdot7}{4\cdot8\cdot12}+\ldots\text{ to infinity.}$$

Or

(*b*) Sum to n terms the series:—

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots\ldots$$

5. Show that the total number of combinations of n things taken any number at a time when the things are not all different is $(p+1)\,(q+1)\,(r+1)\,\ldots\,-1$, where $p, q, r\,\ldots$ represent the number of each kind of thing.

Hence find how many integral factors other than unity the number 1680 has.

6. The co-ordinates of the vertices of a quadrilateral are (1, 3), (3, 9), (5, 7), (9, −1). Find the equations of the lines joining the mid-points of opposite sides, and that of the line joining the mid-points of the diagonals. Show that these lines are concurrent, and find their point of intersection.

7. The co-ordinates of two of the vertices of a triangle are (3, 5) and (7, 18). The co-ordinates of the orthocentre are (93, −25). Find the co-ordinates of the remaining vertex.

8. Show that for different values of n the point

$$\left(\frac{x_1 + nx_2}{1+n},\ \frac{y_1 + ny_2}{1+n}\right)$$

will represent any point in the line joining (x_1, y_1) to (x_2, y_2).

Find the equation of a circle touching the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ at the point (0, 6) and also touching the x-axis.

9. The parabolic path of a stone thrown into the air, referred to horizontal and vertical axes through the point of projection is given by the equation

$$y = x \tan \alpha - \frac{gx^2}{2u^2\cos^2\alpha}.$$

Find (*a*) the co-ordinates of the vertex.

(*b*) the equation of the axis.

(*c*) the co-ordinates of the focus.

(*d*) the equation of the directrix.