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LEAVING CERTIFICATE EXAMINATION, 1925

HONOURS
MATHEMATICS (I).

WEDNESDAY, 17th JUNE.—10 A.M *to* 1 P.M.

[Tables of Measures, Constants, and Formulae may be obtained from the Superintendent.]

1. Solve the equations, giving answers to two places of decimals :—

(a)
$$y^2 + yz + z^2 = 7,$$
$$z^2 + zx + x^2 = 12,$$
$$x^2 + xy + y^2 = 13 ;$$

(b) $\quad 5\left(x + \sqrt{x^2 - 1}\right)^2 + 7\left(x - \sqrt{x^2 - 1}\right)^2 = 12.$

2. Find the sum to n terms of a geometrical progression. Under what circumstances is the series convergent?

(a) Show that if $u_n < ar^n$, $(r^2 < 1)$ for all values of n, the series $u_1, u_2, u_3, \ldots$ etc., is convergent.

(b) Show that
$$1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots \text{ is convergent.}$$

3. State the Binomial Theorem.

(a) Find the cube of
$$1 + \frac{1}{3}x + \frac{1 \cdot 4}{3 \cdot 6}x^2 + \frac{1 \cdot 4 \cdot 7}{3 \cdot 6 \cdot 9}x^3 + \ldots \text{ to infinity,}$$

when $x = \dfrac{19}{27}$.

4. Find the limits of the function

$$\frac{x^2 - x + 1}{x^2 + x + 1} \left(= y \right).$$

Make a rough diagram to show the variation of the function. Find accurately the angle at which the curve cuts the y axis.

5. A man leaves property worth £343 a year to his wife for life; on her death the property is to pass to her nephew for his life. According to the insurance tables it is found that the wife may be expected to live 12 years and the nephew 34 years. What is the present value of the annuity to the nephew, compound interest being reckoned at $4\frac{1}{2}$ per cent. per annum?

6. Find the length of the perpendicular from the point (x, y) on the line $ax + by + c = 0$.

Find the equations of the bisectors of the angles between the lines $3x + 4y = 36$ and $12x + 5y = 78$. How do you distinguish between them?

7. The arch of a bridge consists of three circular arcs. One of these has centre at the point $(0, 0)$ and connects the points $(56, 192)$ and $(-56, 192)$; the second has centre at $(14, 48)$ and runs from $(56, 192)$ to $(104, 0)$; the third is symmetrical with the second on the other side of the first. Show that the arcs have common tangents at their points of contact with each other, and calculate the height of the arch at the middle.

8. If $\quad S = x^2 + y^2 + 2gx + 2fy + c = 0,$
$$S_1 = x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0.$$

Assign meanings to $g, f, c, a^2 + b^2 + 2ga + 2fb + c, S - \lambda S = 0.$

(a) Show that for different values of λ,

$$x^2 + y^2 - 2\lambda x + c^2 = 0,$$

represents a system of coaxal circles. Find the limiting points and the equation of the orthogonal system.

9. When a ray of light strikes a mirror of any shape at a point, the ray and its reflection make equal angles with the normal at the point, and lie in the same plane with the normal. Show that if a point source of light is placed at the focus of a parabola, the reflected rays will all be parallel to the axis of the parabola.

(St. lines may be taken to represent rays of light.)