Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2016

Mathematics

Paper 1
Higher Level

Friday 10 June – Afternoon 2:00 to 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner			
Question	Mark	Question	Mark
1		11	
2		12	
3		13	
4		14	
5			
6			
7			
8			
9			
10		Total	

Grade

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 **(Suggested maximum time: 10 minutes)**

(a) A bus company increases the price of all of its tickets by 6%.
Before the increase, the price of a ticket from Cork to Dublin was €17·00.

(i) Find the price of this ticket **after** the increase.

Six months later, the company reduces the price of this ticket back to €17·00.

(ii) Find the **percentage decrease** in the price of this ticket.
Give your answer correct to one decimal place.

(b) Insert brackets into each of the following statements to make them true.
You may need more than one pair of brackets in some of the statements.

(i) $5 + 4 \times 2 + 3 = 45$

(ii) $5 + 4 \times 2 + 3 = 25$

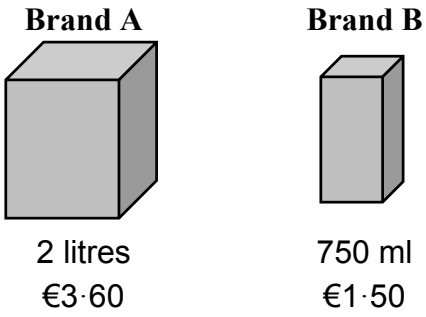
(iii) $5 + 4 \times 2 + 3 = 21$

Question 2 **(Suggested maximum time: 10 minutes)**

(a) **(i)** Write 20 cent as a **fraction** of €20. Give your answer in its simplest form.

(ii) Write 0·5 cm as a **fraction** of 2 m. Give your answer in its simplest form.

(b) A shop sells two brands of orange juice, **Brand A** and **Brand B**, as shown.



(i) Find which brand, **A** or **B**, is cheaper per litre.

Show all of your working out.

Samantha needs to buy **at least 5 litres** of orange juice.

(ii) Find the **lowest price** that she could pay to do this, by buying **Brand A**, **Brand B**, or a **combination** of both. Justify your answer fully.

Lowest price =

Question 3 **(Suggested maximum time: 5 minutes)**

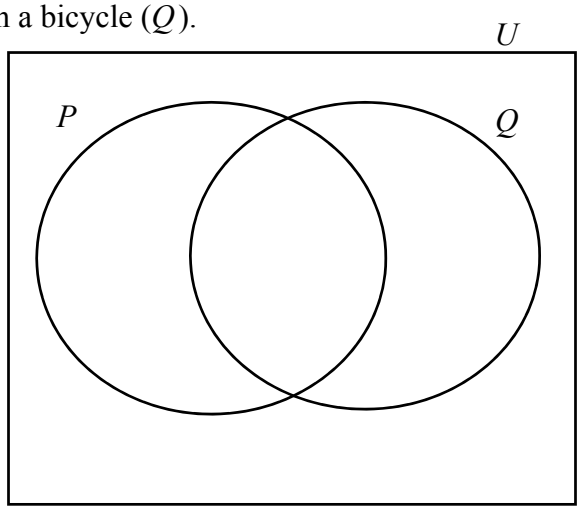
Conor carries out a survey on all of the 25 students in his class (U).
He asks each student if they own a pet (P), and if they own a bicycle (Q).

6 students own **neither** a pet **nor** a bicycle.

28% of the students own **both** a pet and a bicycle.

The ratio $\#(P \setminus Q) : \#(Q \setminus P) = 2 : 1$.

Use this information to fill in the Venn diagram.



Question 4 **(Suggested maximum time: 5 minutes)**

Put a tick (✓) in the correct box in each row of the table below to show whether each statement is always true, sometimes true, or never true, for three **different** sets A, B, and C.

Statement	Tick **one** box only, for each statement		
	Always true	**Sometimes true**	**Never true**
$A \cap B = B \cap A$			
$A \cup B = B \cup C$			
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$			
$A \cup C = A \cap C$			
$A \setminus B = \{\ \}$			

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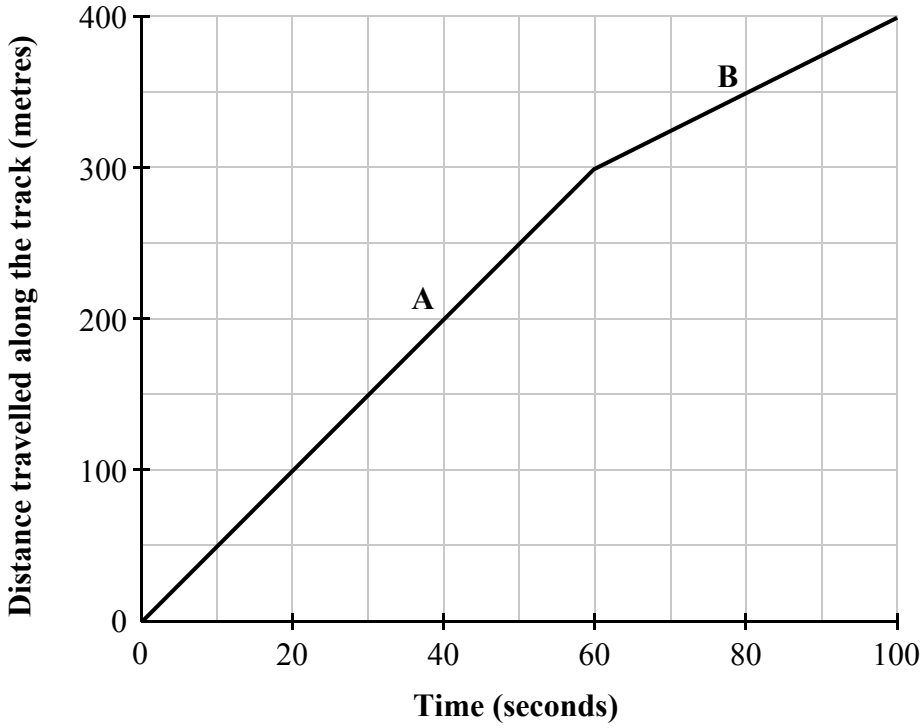
Question 5 **(Suggested maximum time: 10 minutes)**

(a) The graph below shows the distance travelled along a track by Ann over the course of a race. The graph is in two sections, labelled **A** and **B**.



(i) Show that Ann's speed in section **A** is 5 metres per second.

(ii) Find Ann's speed in section **B**, in metres per second.

(b) **Table 1** shows graphs of the **distance** travelled along the track by Bill, Claire, and Dee during the same race. Each person's name is written next to their graph.

Table 2 shows graphs of the **speed** of Bill, Claire, and Dee during the race.
Complete Table 2, by writing the correct name next to each graph.

Table 1

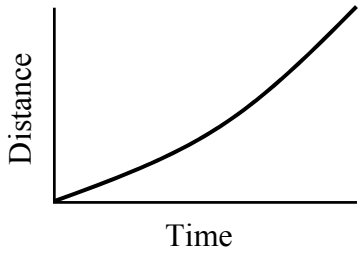
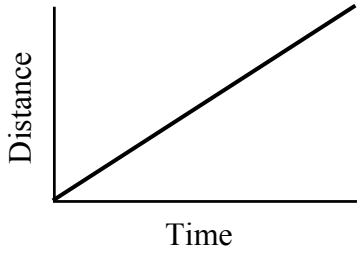
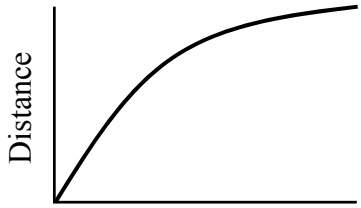
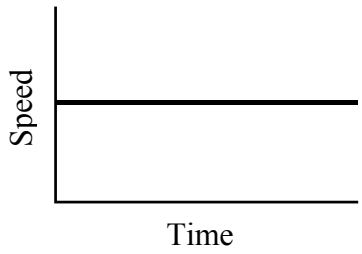
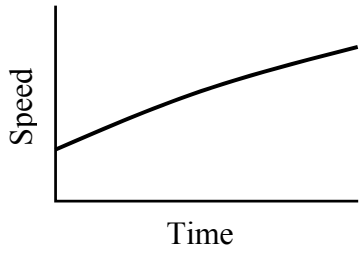
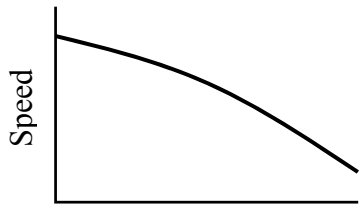
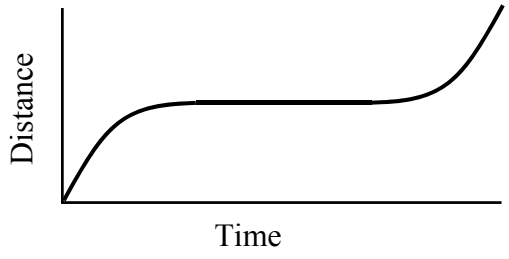
Distance travelled during the race	**Name**
	Bill
	Claire
	Dee

Table 2

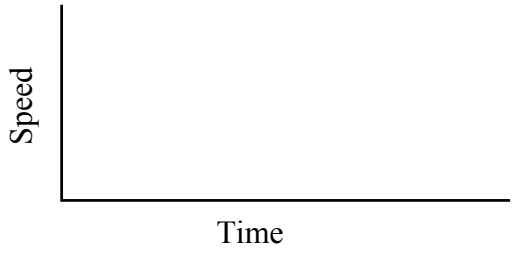
Speed during the race	**Name**
	
	
	

(c) The graph below shows the distance Erik travelled along the track during the same race.
Sketch the graph of Erik's **speed** during the race on the axes below.

Distance travelled by Erik during the race



Erik's **speed** during the race



Question 6 **(Suggested maximum time: 10 minutes)**

(a) Write the following four numbers in order, from the smallest to the biggest.

$\frac{22}{7}$ π $\sqrt{10}$ $3{\cdot}14$

(b) Put a tick (✓) in the correct box in each row of the table below to show whether each number is **rational** or **irrational**. Give a **reason** for each answer.

Number	Tick **one** box only, for each number		**Reason**
	Rational	**Irrational**	
$\sqrt{10}$			
$3{\cdot}14$			

(c) How many digits does the number $3{\cdot}14 \times 10^{100}$ have, when it is written out fully? Justify your answer.

Answer:

Justification:

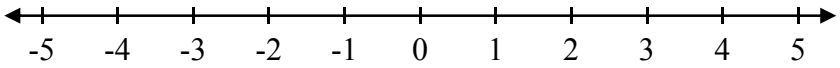
Question 7 **(Suggested maximum time: 5 minutes)**

(a) Solve the following equation.

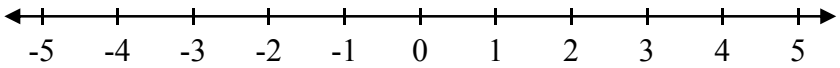
$$\frac{2x+4}{3} - \frac{5x-7}{2} = 5$$

(b) Graph each of the following inequalities on the number line given.

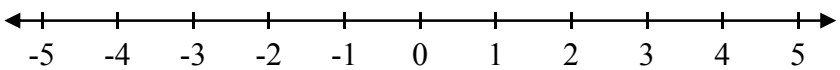
(i) $x < 4$, where $x \in \mathbb{N}$.



(ii) $x < 4$, where $x \in \mathbb{Z}$.



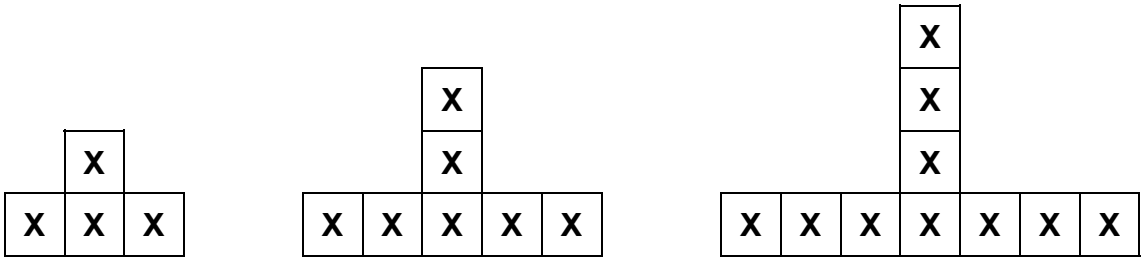
(iii) $x < 4$, where $x \in \mathbb{R}$.



Question 8 **(Suggested maximum time: 15 minutes)**

John makes a sequence where each stage is made up of a certain number of **X**s arranged in a pattern. The first three stages of John's sequence are shown below.

The sequence starts at **stage 0**.



Stage 0 **Stage 1** **Stage 2**

(a) Draw the next stage of John's sequence.

(b) Using a table, a graph, or otherwise, write a **formula** to express N in terms of S, where N is the number of **X**s in stage S of John's sequence.

(c) There are exactly 130 **X**s in stage k of John's sequence. Find the value of k.

(d) Yoko is also making a sequence, with each stage made up of a certain number of **X**s arranged in a pattern. In Yoko's sequence, the relationship between N and S is given by the formula:

$$N = 1 + 2S,$$

where N is the number of **X**s in stage S of the sequence (starting at stage 0).

(i) **Draw** one possible example of the first three stages of Yoko's sequence in the table below.

Yoko's sequence		
Stage 0	**Stage 1**	**Stage 2**

(ii) p represents the number of **X**s in stage y of Yoko's sequence.

Write down the number of **X**s in stage $y + 3$ of Yoko's sequence.
Give your answer in terms of p.

Question 9 **(Suggested maximum time: 5 minutes)**

(a) Write each of the following numbers in the form 3^k, where $k \in \mathbb{Q}$.

(i) 9

(ii) 1

(iii) $\sqrt{27}$

(iv) $\frac{1}{\sqrt[3]{3}}$

(b) Write $(-2n)^4$ in the form an^b, where $a, b \in \mathbb{Z}$.

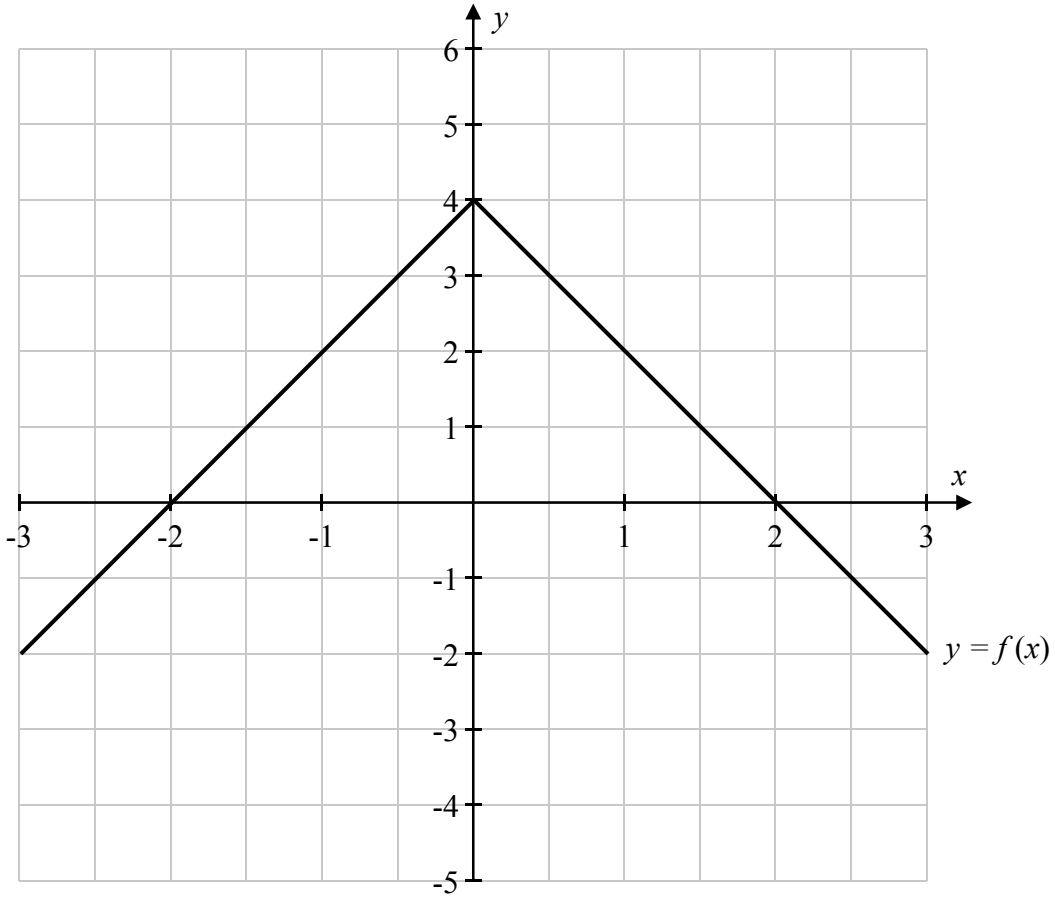
(c) x and $\sqrt{x^2}$ are **not** always equal.

Give an example of a value of x, and the corresponding value of $\sqrt{x^2}$, which are **not** equal.

$x =$ $\sqrt{x^2} =$

Question 10 **(Suggested maximum time: 15 minutes)**

(a) The graph of the function $y = f(x)$ is shown on the co-ordinate diagram below, for $-3 \leq x \leq 3$, $x \in \mathbb{R}$. The graph is made up of two line segments.

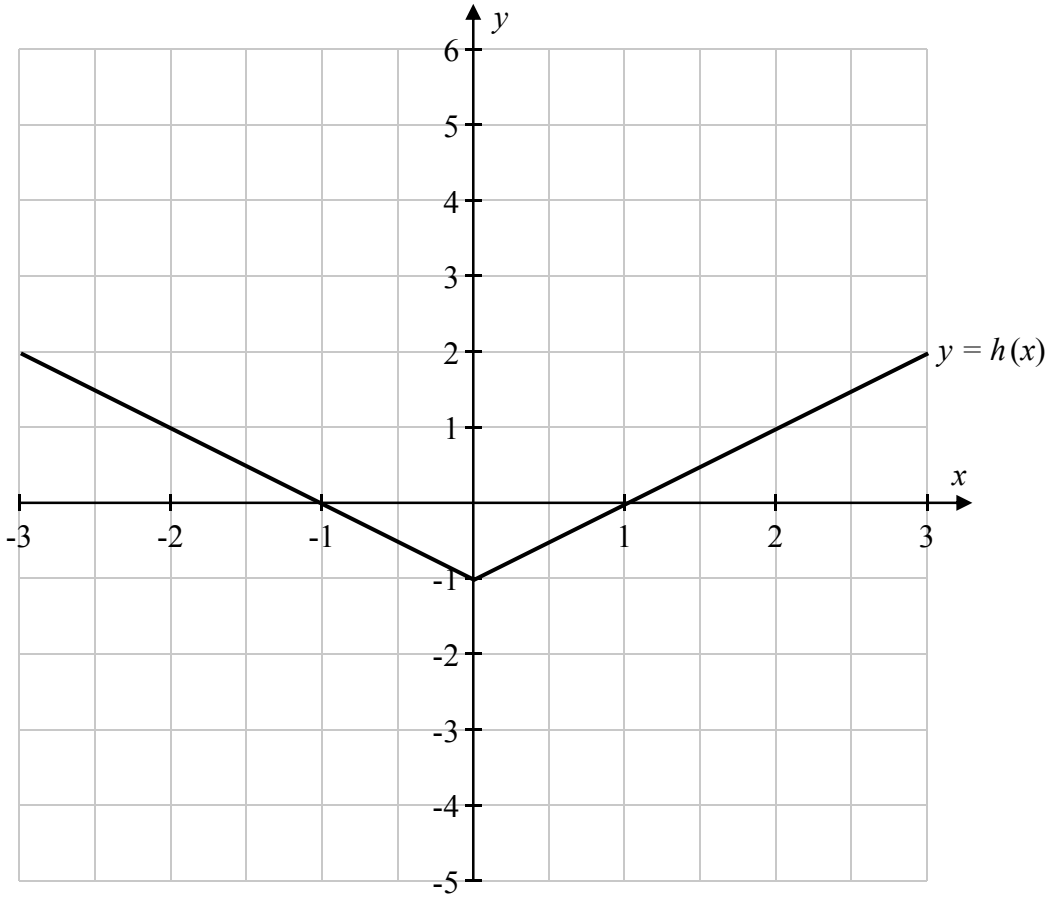


(i) Fill in the table below to show the value of $\boldsymbol{f(x)}$ and the value of $\boldsymbol{f(x) - 2}$ for each of the given values of x.

$\boldsymbol{x}$	-3	-2	-1	0	1	2	3
$\boldsymbol{f(x)}$							
$\boldsymbol{f(x) - 2}$							

(ii) Hence, or otherwise, **draw** the graph of $\boldsymbol{y = f(x) - 2}$ on the co-ordinate diagram above, for $-3 \leq x \leq 3$, $x \in \mathbb{R}$.

(b) The graph of a different function, $y = h(x)$, is shown on the co-ordinate diagram below, for $-3 \leq x \leq 3$, $x \in \mathbb{R}$. The graph is made up of two line segments.



(i) Fill in the table below to show the value of $\boldsymbol{h(x)}$ for each of the given values of x.

$\boldsymbol{x}$	-3	-2	-1	0	1	2	3
$\boldsymbol{h(x)}$							

(ii) Hence, or otherwise, **draw** the graph of $\boldsymbol{y = [h(x)]^2}$ on the co-ordinate diagram above, for $-3 \leq x \leq 3$, $x \in \mathbb{R}$.

Question 11 **(Suggested maximum time: 10 minutes)**

(a) **(i)** Multiply out and simplify $(x+5)^2$.

(ii) Hence, or otherwise, show that the following expression is always divisible by 4.

$$(x+5)^2 - (x-5)^2$$

(b) Factorise each of the following expressions.

(i) $25x^2 - 49n^2$

(ii) $2x^2 - 9x - 18$

Question 12 **(Suggested maximum time: 10 minutes)**

Three bags are shown in the table below. The mass of each bag (in kg) is also shown.

Bag			
Mass, in kg ($y \in \mathbb{R}$)	$y + 5$	19	$2y^2 + 1$

Two of the bags have the same mass (in kg).

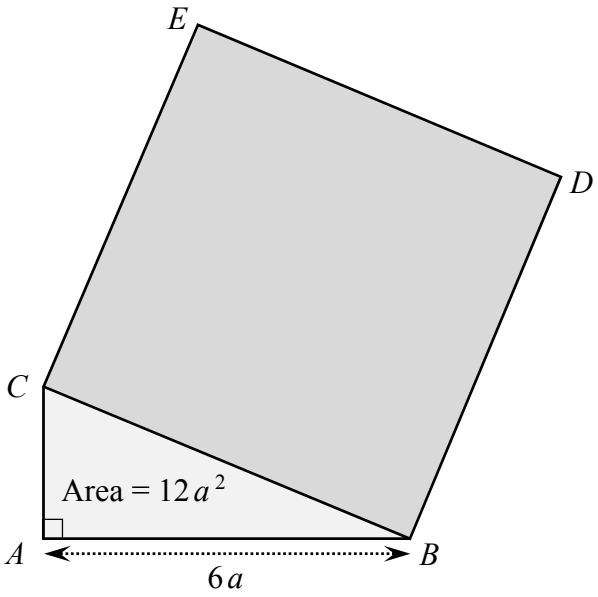
(a) Find the **three** possible positive values of y.
Give your irrational answer correct to two decimal places.

(b) Explain why all three bags can **not** have the same mass (in kg).

Question 13 **(Suggested maximum time: 10 minutes)**

The right-angled triangle ABC is shown in the diagram below.
The square $BDEC$ is placed on the hypotenuse of this triangle.

The **area** of the **triangle** ABC is $12a^2$ square units, where $a \in \mathbb{R}$.
The **length** of the side $[AB]$ is $6a$ units.



Find the **area** of the **square** $BDEC$, in terms of a^2.

Question 14 **(Suggested maximum time: 20 minutes)**

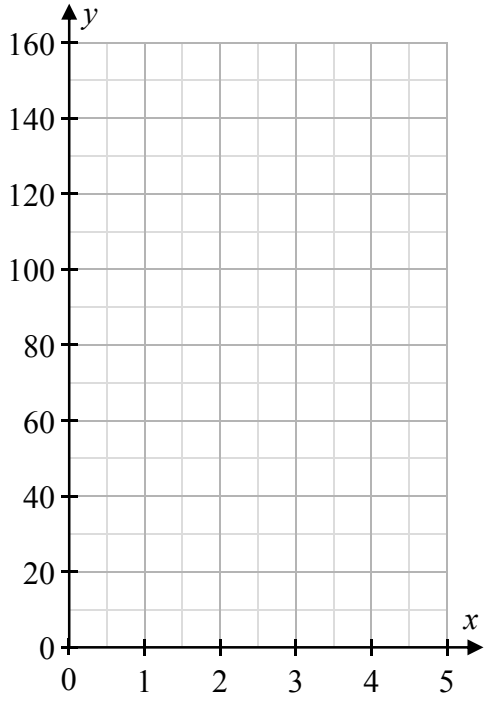
The function $h(x)$ below gives the approximate height of the water at Howth Harbour on a particular day, from 12 noon to 5 p.m.

$$h(x) = 10x^2 - 50x + 130,$$

where $h(x)$ is the height of the water in centimetres, and x is the time in hours after 12 noon.

Source: www.theirishlandscape.com. Altered.

(a) Draw the graph of the function $h(x) = 10x^2 - 50x + 130$ on the axes below, for $0 \le x \le 5,\ x \in \mathbb{R}$.



(b) Use your graph in part **(a)** to answer the following questions.

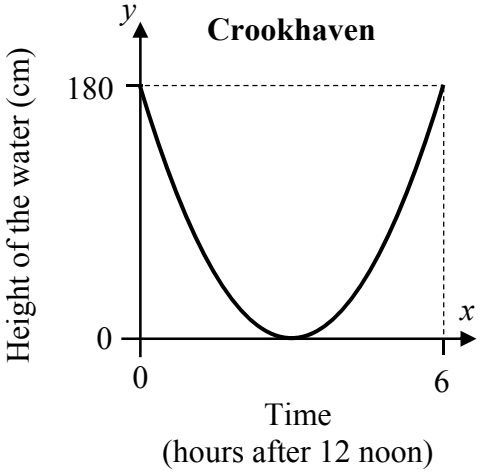
(i) Find the height of the water at 12 noon.

(ii) Estimate the height of the water at its lowest point.

(iii) After 12 noon, how long did it take before the water was at its lowest point?

The graph on the right shows the approximate height of the water in centimetres at Crookhaven on a different day, from 12 noon to 6 p.m. The graph is symmetrical.

On this day, the height of the water at 12 noon was 180 cm, and the height of the water at the lowest point on the graph was 0 cm.



(c) Taking x as the time in hours after 12 noon, this graph is given by the function

$$g(x) = ax^2 + bx + c,$$

where $a, b, c \in \mathbb{Z}$, and $x \in \mathbb{R}$.

(i) Find the value of c.

(ii) Hence, or otherwise, find the value of a and the value of b.

Junior Certificate 2016 – Higher Level

Mathematics – Paper 1

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