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JUNIOR CERTIFICATE EXAMINATION, 2000

MATHEMATICS - HIGHER LEVEL

THURSDAY, 8 JUNE - MORNING, 9.30 to 12.00

PAPER 1 (300 marks)

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

Marks may be lost if necessary work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

1. **(i)** A solicitor's fee for the sale of a house is 2 % of the selling price. If the fee is IR£1320, calculate the selling price of the house.

(ii) A distance of 600 metres is travelled in 30 seconds. Find the average speed in km/hr.

(iii) The radius of the base of a cone and the radius of a sphere are each 4 cm in length. The volume of the cone is equal to the volume of the sphere. Find the height of the cone.

(iv) Evaluate $\left(\frac{1}{0.25}\right)^2 - \sqrt{72.25}$.

(v) $f: x \to 1 + 4x$ and $g: x \to 1 - 4x$ are two functions defined on $\mathbf{R}$. Find $(f \circ g)(2)$.

(vi) $A = \{1,2,3\}$ and $B = \{2,3,4\}$. Write down the elements of $A \Delta B$ and hence find the set $(A \Delta B) \Delta A$.

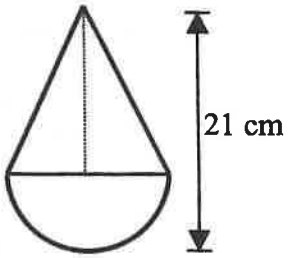
(vii) If $x * y = x - \frac{2}{y}$, find the two values of a for which $a * a = 1$.

(viii) If $\frac{p}{q-p} = q$, express p in terms of q.

(ix) Find the value of $\log_7 35 - \log_7 5$.

(x) Solve the equation $3 - x - 2x^2 = 0$ and hence, or otherwise, find the solution set of the inequality $3 - x - 2x^2 \geq 0 \quad x \in \mathbf{R}$

2. **(a)** A toy is made of a cone which fits exactly on top of a hemisphere, as shown in the diagram. The radius length of the hemisphere is 6 cm and the total height of the toy is 21 cm.

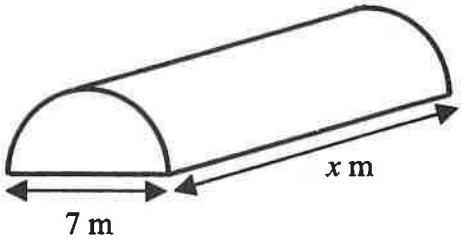


(i) Write down the height of the cone and hence find the volume of the cone in terms of π.

(ii) Find the volume of the hemisphere in terms of π.

(iii) Express the volume of the cone as a percentage of the total volume of the toy. (Give your answer correct to one place of decimals.)

(b) A shed is in the shape of a half-cylinder, closed at both ends, as shown. The diameter length of each semicircular end is 7 m, and the shed is x m long.



(i) Find the area of each semicircular end.

(Take $\pi = \frac{22}{7}$.)

(ii) The total area of the two ends and the roof is 214.5 m^2.
Find x.

3. **(a)** Factorise fully each of the following:

(i) $x^2 - 9x + 20$

(ii) $8x^3 - y^3$

(iii) $9x^2 - (3y - 4x)^2$

(b) Solve for x:

$$\frac{1}{(x-5)(x+3)} - \frac{3}{x-5} = \frac{2}{x+3}, \quad x \neq 5, x \neq -3.$$

(c) **(i)** Solve, correct to one decimal place, the equation

$$2x^2 - 3x - 4 = 0.$$

(ii) Using your answers to part **(i)**, or otherwise, find, correct to one decimal place, the two values of a for which

$$2(a+3)^2 - 3(a+3) - 4 = 0.$$

4. **(a)** The following table shows the distribution of the time in minutes taken by a group of 40 students to complete a test:

Time in minutes	0 – 10	10 – 30	30 – 40	40 – 60	60 – 100
Number of students	4	6	6	8	16

(Note: 0 – 10 means 0 or more but less than 10, etc.)

(i) Draw a histogram to illustrate the above data.

(ii) Copy and complete the following cumulative frequency table:

Time in minutes	< 10	< 30	< 40	< 60	< 100
Number of students					

(iii) On graph paper, draw the ogive (cumulative frequency curve), putting the number of students on the vertical axis.

(iv) Use your graph to estimate the interquartile range.

(b) The mean of the following frequency distribution is 13. Find the value of x.

Number	10	12	14	16
Frequency	5	6	x	4

5. **(a)** Using the same axes and scales, draw the graphs of

$f: x \to 3 + x - 2x^2$

$g: x \to 1 - x$

in the domain $-2 \leq x \leq 3$, $x \in \mathbf{R}$.

Use your graphs to estimate

(i) the maximum value of $f(x)$

(ii) the range of values of x for which $f(x) \geq g(x)$.

(b) $h: x \to 2x - a$ and $k: x \to ax + b$ are two functions defined on $\mathbf{R}$, where a and $b \in \mathbf{Z}$.

$h(3) = 1$ and $k(5) = 8$.

(i) Find the value of a and the value of b.

(ii) Hence, list the values of x for which $h(x) \geq k(x)$, $x \in \mathbf{N}$.

(a) A survey was taken of a group of 44 students, each of whom was studying one or more of the three subjects History, Geography and Art.
28 students studied History.
30 students studied Geography.
22 students studied Art.
6 students studied History only.
15 students studied both History and Geography.
3 students studied all three subjects.

(i) Use a Venn diagram to find the number of students who studied History and Geography but not Art.

(ii) How many students studied History and Art but not Geography?

(iii) Find the number of students who studied Geography only.
[*Hint*: Let x represent the number of students who studied Geography and Art but not History.]

(b) 210 people attended the first night of a concert. They were seated in rows, each of which contained x people.

(i) Express, in terms of x, the number of rows needed.

(ii) The following night 216 people attended the concert. They were seated in rows, each of which contained $x + 1$ people. Write down an expression in x for the number of rows needed on the second night.

(iii) On the second night, there were 3 fewer rows needed than on the first night. Write down an equation in x to represent this information and use it to calculate x.