

AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

JUNIOR CERTIFICATE EXAMINATION, 1999

MATHEMATICS - HIGHER LEVEL

THURSDAY, 10 JUNE - MORNING, 9.30 to 12.00

PAPER 1 (300 marks)

Attempt QUESTION 1(100 marks) and FOUR other questions (50 marks each).

Marks may be lost if necessary work is not clearly shown.
 Mathematics Tables may be obtained from the Superintendent.

1. (i) A bill for IR£27 includes VAT at $12\frac{1}{2}\%$. Calculate the amount of the bill before VAT is added.
- (ii) A distance of 18 km is travelled in 25 minutes. Find the average speed in m/s.
- (iii) The scale on a map is 1: 25 000. The length of a wall on the map is 2.8 mm. Calculate the actual length in metres.
- (iv) Evaluate
- $$\sqrt{\frac{1}{(0.2)^2} - 4.75}.$$
- (v) If $x = \frac{p+q}{p-q}$, express p in terms of q and x .
- (vi) If $\log_k 8 = \frac{3}{2}$, find the value of k .
- (vii) Write the following as a single fraction in its simplest form:
- $$\frac{4}{2x-3} - \frac{1}{3-2x}, \quad x \neq \frac{3}{2}.$$
- (viii) Express $\sqrt{50} - \sqrt{8}$ in the form $b\sqrt{2}$ where $b \in \mathbb{N}$.

(ix) Express

$$5 \times 10^{-2} + 1.2 \times 10^{-3}$$

in the form $a \times 10^n$ where $1 \leq a < 10$ and $n \in \mathbf{Z}$.

(x) If the solution of the inequality

$$x^2 - 4x + 3 \leq 0, \quad x \in \mathbf{R},$$

is given as $a \leq x \leq b$, find the value of a and the value of b .

2. (a) A cylindrical metal pipe is 3 m long. It has external radius of length 11 cm and internal radius of length 10 cm.

(i) Find, in cm^3 , the volume of metal in the pipe in terms of π .

(ii) A solid cylinder has height 7 cm. Its volume is equal to the volume of metal in the pipe. Calculate its radius length.

(b) The radius of a sphere and the radius of the base of a solid cone are each 5 cm in length.

Write down the surface area of the sphere in terms of π .

The total surface area of the cone is equal to the surface area of the sphere. Calculate the slant height of the cone.

Hence, calculate the vertical height of the cone, correct to one place of decimals.

3. (a) Factorise fully each of the following:

(i) $x^2 + x - 12$

(ii) $x^3 + 27y^3$

(iii) $x^3 + 2x^2y - xy^2 - 2y^3$.

(b) Simplify

$$(2x^3 - 7x^2 - 7x + 30) \div (2x - 5).$$

(c) Solve, correct to one decimal place, the equation

$$x^2 = 1 - 4x.$$

4. (a) The number of apples yielded by each of 50 trees in an orchard was recorded and the following cumulative frequency table was drawn up:

Number of apples per tree	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Number of trees	2	6	15	28	45	48	50

On graph paper, draw the ogive (cumulative frequency curve), putting the number of trees on the vertical axis.

Use your graph to estimate

- (i) the number of trees which yielded 25 apples or less
 (ii) the median number of apples per tree
 (iii) the interquartile range.
- (b) The mean of the following frequency distribution table is 4. Find the value of k .

Number	2	$k-2$	k	7	8
Frequency	7	10	$k+3$	4	1

5. (a) Using the same axes and scales, draw the graphs of

$$f: x \rightarrow 9 - 3x - 2x^2$$

$$g: x \rightarrow -x$$

in the domain $-4 \leq x \leq 2, x \in \mathbf{R}$.

Use your graph to estimate

- (i) the maximum value of $f(x)$
 (ii) the values of x for which $f(x) = g(x)$.
- (b) $h: x \rightarrow x^2 + x + q$ is a function defined on \mathbf{R} where $q \in \mathbf{Z}$.
- (i) If $h(-3) = 0$, find the value of q .
 (ii) Hence, solve the equation $h(x + 5) = 0$.

6. (a) A school sports day was attended by 250 pupils.

70 pupils took part in the long jump event.

40 pupils took part in the high jump event.

160 pupils did not take part in either of the two jump events.

Using a Venn diagram, or otherwise, find the number of pupils who took part in the long jump event only.

- (b) Solve the simultaneous equations:

$$2x - y = 18$$

$$\frac{x}{3} - \frac{y}{4} = 2$$

- (c) Rectangular flagstones are to be laid, edge to edge, in a single line along a straight path which is 150 m long. Each flagstone is x m long.

(i) Express, in terms of x , the number of flagstones needed.

(ii) If each flagstone were 0.2 m longer express, in terms of x , the number of flagstones which would then be needed.

(iii) If the longer flagstones were used, the total number required would decrease by 25. Write down an equation in x to represent this information and use it to calculate x .