JUNIOR CERTIFICATE EXAMINATION, 1997

MATHEMATICS – HIGHER LEVEL – PAPER 1 (300 marks)

THURSDAY, 12 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

Marks may be lost if all your work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

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1. (i)   Calculate $2\frac{1}{4}\%$ of IR£164.

    (ii) It takes 4 hours and 20 minutes to travel a journey at an average speed of 120 km/hr. How many hours and minutes will it take to travel the same journey if the average speed is reduced to 100 km/hr?

    (iii) The height of a cylinder is equal to the length of its diameter. The curved surface area of the cylinder is 100π cm². Calculate the height.

    (iv) Solve, correct to one place of decimals, the equation

    $$x^2 - 2x - 1 = 0.$$

    (v)   Evaluate

    $$\sqrt{\frac{7}{0.5} + (2.5)^2}\,.$$

    (vi) If $f : x \rightarrow x^2 + 2x - 1$, find the value of $f\left(\frac{3}{5}\right)$.

    (vii) Find the value of n if

    $$\frac{2^{2n+3}}{2^{n+1}} = 32\,.$$

    (viii) Calculate the value of k if
    $$k = \log_2 64 - \frac{1}{2}\log_2 16.$$

    (ix) If $x * y = 2x + 3y$, find the value of a for which

    $$3 * (2 * a) = 54.$$

    (x)   Show on the number line the range of values of x for which

    $$-3 \le 2x - 1 \le 7, \quad x \in \mathbf{R}.$$

2. (*a*) An empty rectangular oil tank has an internal length of 150 cm and an internal width of 50 cm. The price of oil is 24p per litre. A delivery of oil costing IR£144 fills half the capacity (internal volume) of the tank.

Calculate

(*i*) the number of litres of oil required to fill the tank completely

(*ii*) the internal height of the tank. [1 litre = 1000 cm³]

(*b*) Calculate, in terms of π, the volume of a sphere with radius of length 3 cm.

A cylinder of height 9 cm has a volume equal to 4 times the volume of the sphere. Calculate the length of the radius of the cylinder.

The cylinder is partly filled with water. Nine identical solid cubes with edge of length 2 cm are then submerged in the water. The water rises in the cylinder but it does not overflow. How far does the water rise? Give your answer correct to one place of decimals. Take $\pi = \dfrac{22}{7}$.

3. (*a*) Factorise each of the following:

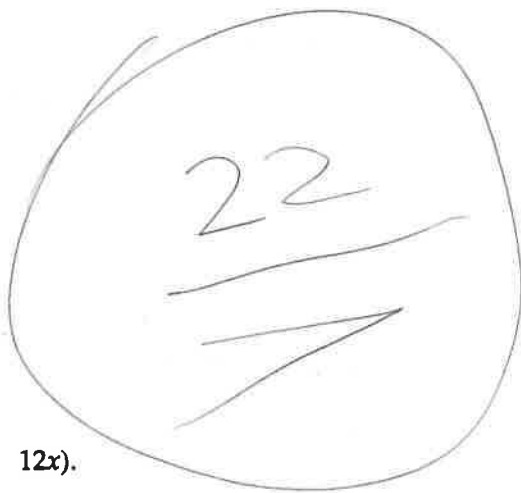(*i*) $15bc - 3c^2$ 439

(*ii*) $an - 5a - 5b + bn$

(*iii*) $x^3 - 8y^3$.

(*b*) Simplify and, hence, factorise

$$(3x - 2y)^2 - y(5y - 12x).$$

(*c*) Solve for x:

(*i*) $5 = \dfrac{6}{x} + \dfrac{8}{x^2}, \quad x \neq 0$

(*ii*) $\dfrac{3}{x - 1} - \dfrac{2}{x + 1} = 1, \quad x \neq 1, \ x \neq -1.$

4. **(a)** The following cumulative frequency table shows the results of a test given to a group of 50 students:

Mark	≤ 20	≤ 50	≤ 70	≤ 80	≤ 90	≤ 100
No. of Students	4	22	38	46	48	50

    (i) Draw the cumulative frequency curve (ogive), putting the number of students on the vertical axis.

    (ii) Estimate the median mark.

    (iii) Use the cumulative frequency curve to estimate the number of students who scored between 40 marks and 85 marks.

    (iv) If 60% of the students passed this test, estimate the lowest mark that a student who passed the test could have obtained.

(b) People attending a course were asked to choose one of the whole numbers from 1 to 12. The results were recorded as follows:

Number	1 – 3	4 – 6	7 – 9	10 – 12
No. of People	3	x	2	8

Using mid-interval values, 6.5 was calculated as the mean of the numbers chosen.

Find the value of x.

5. **(a)** Using the same axes and the same scales, graph the functions

$$f : x \rightarrow 3x^2 - 2x - 5$$
$$g : x \rightarrow 4 - 2x$$

in the domain $-2 \leq x \leq 3$, $x \, \varepsilon \, \mathbf{R}$.

Estimate from your graph the range of values of x for which $f(x) \leq g(x)$.

(b) $h : x \rightarrow 2x + a$ and $k : x \rightarrow b - 5x$ are two functions defined on $\mathbf{R}$, where a and b are real numbers.

    (i) If $h(1) = -5$ and $k(-1) = 4$, find the value of a and the value of b.

    (ii) Find, in terms of x, an expression for $(hok)(x)$.

    (iii) Find the values of x for which

$$(hok)(x) = x^2.$$

6.   *(a)*   Solve the simultaneous equations

$$3x + y = 19.5$$
$$x - 2y = 3.$$

(b)   *(i)*   If $\{2, 4, 6\}$ Δ $\{2, 3\}$ = A, find the set A.

     (ii)   If $\{1, 3, 4\}$ = B Δ $\{2, 4, 6\}$, find the set B.

(c)   A closed rectangular box has a square base of side x cm. The height of the box is 5 cm.

The total surface area of the box is 288 cm².

Write down an equation in x to represent this information and use it to calculate x.

5 cm

x cm

x cm