AN ROINN OIDEACHAIS

JUNIOR CERTIFICATE EXAMINATION, 1996

MATHEMATICS — HIGHER LEVEL — PAPER 2 (300 marks)

FRIDAY, 7 JUNE — MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

Marks may be lost if necessary work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

1. **(i)** Each side of a square is increased in length from 5 cm to 7 cm. Calculate the percentage increase in the area of the square.

(ii) If

$$2^{-3} + 3^{-2} = \frac{p}{q}, \quad p, q \in \mathbf{N_0},$$

find the value of p and the value of q.

(iii) Express

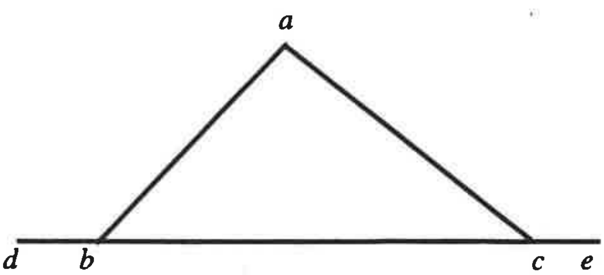
$$8.425 \times 10^2 - 42 \times 10^{-1}$$

in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbf{Z}$.

(iv) abc is a triangle with the line bc containing the points d and e.

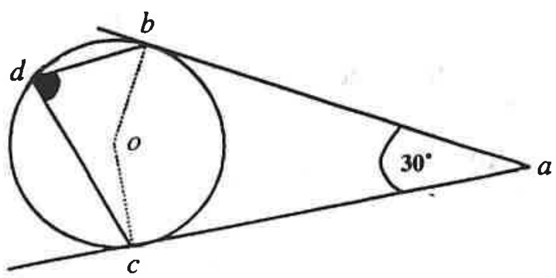
Prove that

$|\angle abd| + |\angle ace| > 180°$.



(v) ab and ac are tangents to the circle at b and c, respectively. The centre of the circle is o and d is a point on the circle.

If $|\angle bac| = 30°$, find $|\angle bdc|$.



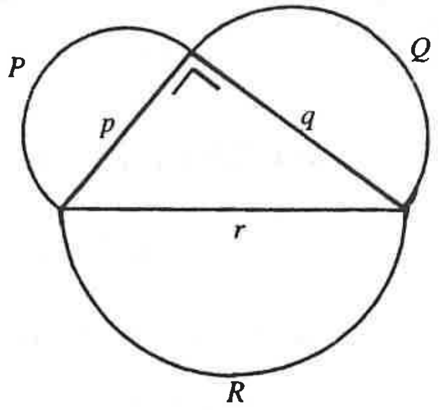
(vi) The lengths of the sides of a right angled triangle are p, q and r as shown.

P, Q and R are semicircles with diameters of length p, q and r, respectively.

Express the area of P in terms of p and π.

Prove

$$\text{area of } P + \text{area of } Q = \text{area of } R.$$



(vii) Find the image of the point (1,–2) under the central symmetry in the point (–2,2).

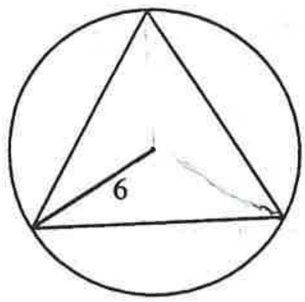
(viii) The equation of the line K is $y + x = 0$. The line L is the image of K under the translation

$$(0,0) \rightarrow (4,-3).$$

Find the equation of L.

(ix) A triangle has three sides of equal length. Its circumcircle has radius of length 6.

Find the length of a side of the triangle.



(x) If $\tan A = \frac{5}{4}$, $0^\circ \leq A < 90^\circ$, find the value of

$$\sin A + \cos A$$

without using the Tables.

2. (a) A sum of money, IR£5900, was invested for one year. It earned interest at a rate of 8% per annum. Calculate the amount of the investment at the end of the year.

A charge of IR£x was then deducted from this amount. The money which remained was converted into dollars ($) and the dollars were invested for a year at a rate of interest of 9% per annum.

At the end of the year, the invested dollars amounted to $10 137.

If the exchange rate was IR£1 = $1.50 on the day the punts (IR£) were changed to dollars, calculate x.

(b) If

$$x^2y + 2xy - 1 = 0$$

express y in terms of x.

Find the value of a for which

$$\frac{1}{y} = (x + a)^2 - a, \quad a \in \mathbf{N}.$$

3. Prove that the opposite angles and the opposite sides of a parallelogram are equal in measure.

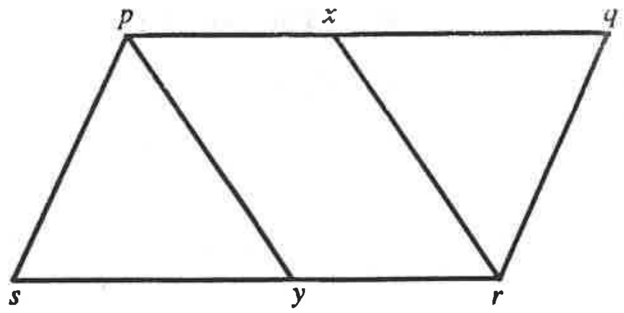
psrq is a parallelogram.
py and *rx* bisect $\angle spq$ and $\angle srq$, respectively.

Prove

$$|sp| = |sy|.$$

Deduce

$$|px| = |yr|.$$



4. Prove that if the angles of two triangles are, respectively, equal in measure, then the lengths of the corresponding sides are proportional.

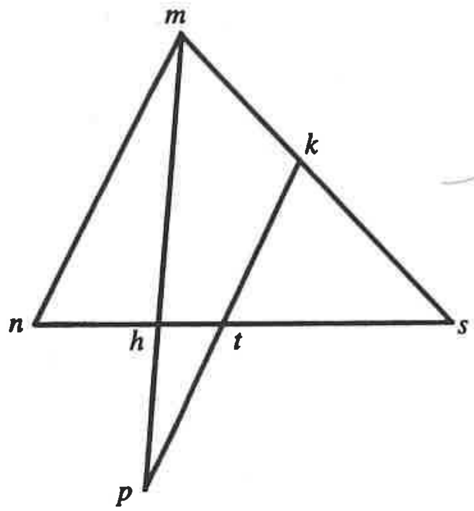
kp is parallel to *mn* and *t* is the midpoint of [*kp*].

Prove

$$\frac{|mn|}{|tk|} = \frac{|nh|}{|ht|}.$$

Deduce

$$\frac{|ns|}{|ts|} = \frac{|nh|}{|ht|}.$$



5. **(a)** The equation of the line L is $2x + 3y = k$.
The point $p\ (-2,1)$ is on L.

Find

(i) the value of k

(ii) the slope of L

(iii) the coordinates of the point of intersection of L and the y-axis

(iv) the equation of the line K through p perpendicular to L

(v) the area of the triangular region enclosed by L, K and the y-axis.
(Graph paper is available from the Superintendent).

(b) $a\ (5,2)$, $b\ (2,-1)$, $c\ (x,4)$ and $d\ (4,x)$ are four points and $|ab| = |cd|$,

prove that

$$(x - 4)^2 = 9$$

and solve for x.

6. **(a)** If $\cos A = \frac{1}{2}$, for $0° \leq A \leq 90°$,

find

(i) the measure of the angle A

(ii) $\cos \frac{A}{2}$.

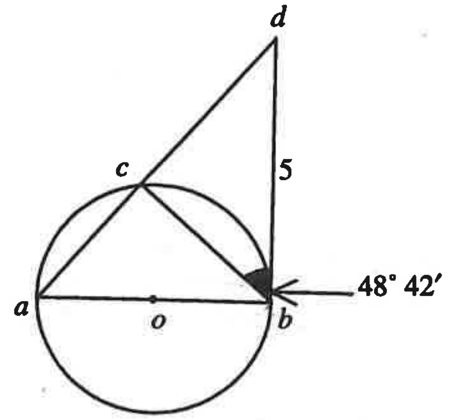
(b) The circle with centre o contains the points a, b and c.

db is a tangent to the circle at b.

$|\angle cbd| = 48° 42'$ and $|db| = 5$.

(i) Calculate $|cb|$, as accurately as the Tables allow.

(ii) Find $|ab|$, correct to the nearest whole number.



(c) pqr is a triangle such that

$$|pq| = |qr| = |rp|.$$

(i) If the area of triangle pqr is 6.928 square units, find $|pq|$ correct to the nearest whole number.

(ii) If $|\angle spq| = 50°$ and $|sr| = 10$, calculate $|\angle psr|$ correct to the nearest degree.