JUNIOR CERTIFICATE EXAMINATION, 1996

MATHEMATICS — HIGHER LEVEL — PAPER 1 (300 marks)

THURSDAY, 6 JUNE — MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

Marks may be lost if necessary work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

1. **(i)** A distance of 375 metres is travelled in 25 seconds.

   Find the average speed in kilometres per hour.

   (ii) By selling a car for IR£8840 a loss of 35% is made on the purchase price.

   Calculate the purchase price.

   (iii) A sphere with radius of length 3 cm has a volume equal to eight times the volume of a sphere with radius of length r cm. Calculate r.

   (iv) If $A = \{1,2,3\}$, $B = \{2,3,4\}$ and $C = \{3,4,5\}$, find the set

   $$A \triangle B \triangle C.$$

   (v) If $f: x \rightarrow \dfrac{x}{3} - 2$, what is $f^{-1}(-5)$?

   (vi) Solve for x:

   $$3(x - 1)^2 - 2(x - 1) - 1 = 0.$$

   (vii) If $\dfrac{1}{b} = \dfrac{3}{p} - \dfrac{4}{a}$, express p in terms of a and b.

   (viii) If $x * y = x(3y - 5)$, calculate the value of $3 * (5 * 2)$.

   (ix) If $27^{-\frac{1}{3}} = x$, find the value of x.

   (x) Using the graph of a quadratic function, or otherwise, find the solution set of the inequality
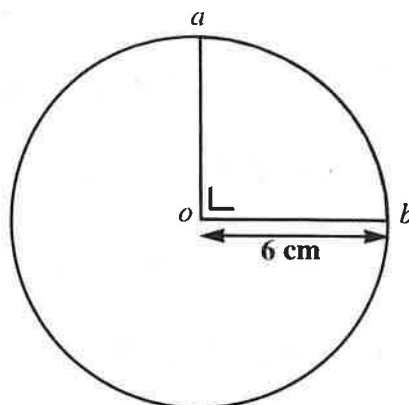
   $$6 - x - x^2 \geq 0, \quad x \in \mathbf{R}.$$

2. (a) (i) A circle with centre o and radius of length 6 cm contains the points a and b.

Find the area of the shaded region if $|\angle aob| = 90°$.

Give your answer in terms of π.

(ii) Calculate the total length of the perimeter of the shaded region.

(Take $\pi = 3.14$).

(b) Wax in the shape of a cylinder with radius of length 4 cm and height 36 cm is melted down. The resulting wax is formed into cone shaped candles. Each candle has height 6 cm and base of radius length 2 cm.

(i) Calculate the number of candles that can be made, assuming that no wax is lost.

(ii) The candles are placed, base down and in rows of three, in the smallest possible rectangular box. Calculate, in cm^3, the volume (internal capacity) of the box.

(iii) What percentage of the volume of the box is empty?   Take $\pi = 3.14$. Give your answer correct to the nearest whole number.

3. (a) Solve for x:

$$\frac{4}{(x-2)(x+1)} - \frac{2}{x+1} = \frac{3}{x-2}, \quad x \neq 2, \ x \neq -1.$$

(b) Factorise each of the following:

(i) $3x^2 + 2x - 8$

(ii) $2x^2y - 2xz - 3xy + 3z$

(iii) $9a^2 - 12ab + 4b^2 - 16c^2$.

(c) Solve, correct to 1 place of decimals, the equation

$$5x^2 - 7x - 10 = 0.$$

Hence, or otherwise, find the values of y for which

$$5\left(\frac{1}{y}\right)^2 - 7\left(\frac{1}{y}\right) - 10 = 0$$

and give your answers correct to 1 place of decimals.

4. The table below gives a summary of the amount of money, in IR£, spent by 90 people at a charity auction:

Amount of money (IR£)	0-10	10-20	20-30	30-50	50-70	70-100
Number of people	26	18	15	12	10	9

(Note: 0-10 means 0 or more but less than 10, etc.)

   (i) Draw a histogram to illustrate the data.

      (Graph paper is available from the Superintendent).

  (ii) Using mid-interval values, estimate the mean amount spent per person.

      Give your answer correct to the nearest IR£.

(iii) Copy and complete the cumulative frequency table:

Amount of money (IR£)	<10	<20	<30	<50	<70	<100
Number of people		44				

  (iv) Draw the cumulative frequency curve (ogive) from the table in (iii), putting the number of people on the vertical axis.

   (v) Use the cumulative frequency curve to estimate the interquartile range.

5.   **(a)** Graph the function $f: x \rightarrow 9 + 3x - 2x^2$ in the domain $-2 \leq x \leq 4$, $x \in \mathbf{R}$.

      Estimate from your graph

     (i) the value of $f(1.6)$

    (ii) the roots of the equation $2x^2 - 3x - 10 = 0$

   (iii) the range of values of x for which

$$7 + 3x - 2x^2 \geq 0.$$

  (b) $f: x \rightarrow 3x - 1$ and $g: x \rightarrow 3x^2 + 2$ are two functions defined on **R**.

     (i) Find expressions in x for $(f \circ g)(x)$ and for $(g \circ f)(x)$.

    (ii) Find the two values of x for which

$$(f \circ g)(x) = (g \circ f)(x).$$

6. **(a)** Solve the simultaneous equations

$$3x - 2y = y - 6x$$

$$\frac{5x - 3y + 2}{2} = \frac{x - 2y + 4}{3}.$$

(b) If $\log_a 2 = p$ and $\log_a 3 = q$, express, in terms of p and q,

    (i) $\log_a 12$

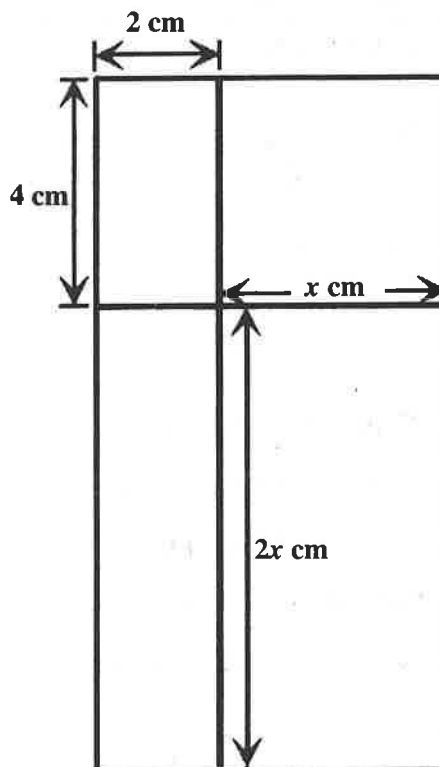   **(ii)** $\log_a \dfrac{9}{4}$.

$$\frac{5x - 3y + 2}{2}$$

$$-1 \quad -3$$

(c) A rectangle of area 72 cm² is divided into 4 smaller rectangles. The lengths of the sides are as shown in the diagram.

Write down an equation in x to represent this information.

Solve for x.

2 cm

4 cm

x cm

$2x$ cm

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