MATHEMATICS – HIGHER LEVEL – PAPER 2 (300 marks)

FRIDAY, 11 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all necessary work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.

1.

(i) A sum of money, invested at 10% per annum interest, amounts to IR£907.50 after one year.
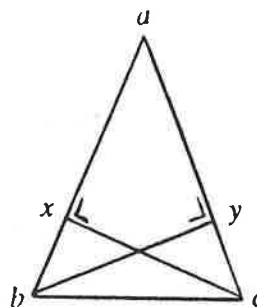How much was invested?

(ii) $x : y : z = 2 : 4 : 5$.
$z = 35$
Find $x + y + z$.

(iii) Simplify $\dfrac{1}{(25)^{\frac{1}{2}}}$.

Give your answer in the form $\dfrac{1}{a}$, $a \in N_0$.

(iv) $a\,b\,c$ is an isosceles triangle
with $|\,ab\,| = |\,ac\,|$.
$cx \perp ab$ and $by \perp ac$.

Write an expression for the area of $\Delta\,abc$.

Hence, or otherwise, prove that $|\,cx\,| = |\,by\,|$

(v) L is the line $y = 1$, M is the line $x = 2$.
Find the image of $p(3 , 2)$ under $S_L \circ S_M$.

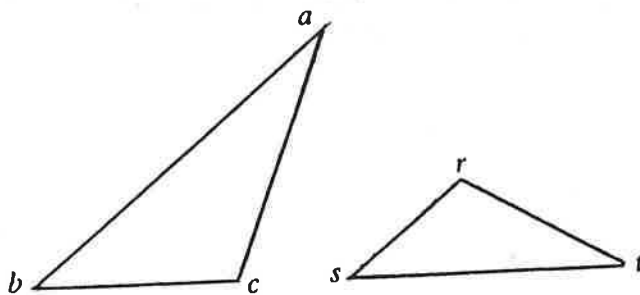Name the single transformation which is equivalent to $S_L \circ S_M$.

(vi) $|\angle abc| = |\angle rst| = 40°$
      $|\angle bca| + |\angle str| = 140°$
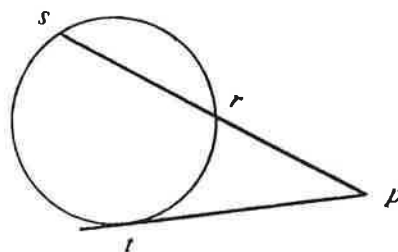
      Show that $|\angle bac| = |\angle str|$

      Hence or otherwise show that

      $$\frac{|ab|}{|st|} = \frac{|bc|}{|rs|}$$

(vii) pt is a tangent to a circle at t.
       ps cuts the circle at r and s.
       $|pr| = |rs|$.

       Show that $|ps|^2 = 2|pt|^2$

(viii) Find the coordinates of the point of intersection of the lines

        $$3x + 4y = -1$$

        $$x - 2y = 3.$$

(ix) M is the line $3x + ky = 11$ which contains $(1, 2)$.
      Find the value of k and the coordinates of the point where M cuts the y-axis.

(x) Find the value of A for which $\cos A = -1$, $0° \le A \le 360°$

2. (a) IR£4000 is borrowed for two years.
         Interest for the first year is charged at 15% per annum.
         Calculate the amount owed at the end of the year.

         IR£2500 is then repaid.
         Interest is charged at $x\%$ per annum on the remainder for the second year.
         The amount owed at the end of the second year is IR£2500.
         Calculate x correct to the nearest integer.

    (b) If $\dfrac{1}{h^2} - 8p = m^2$

         express h in terms of p and m.

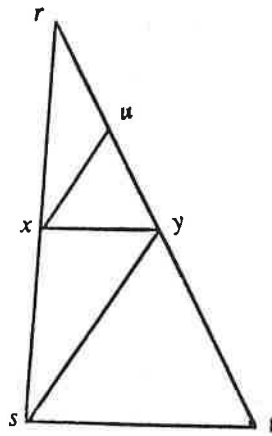   Determine the values of h when $m = 10$ and $p = -8$.

3.   *(i)*   Prove that a line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

(ii)   In $\triangle$ *rst*,   *xy* $\parallel$ *st*,

xu $\parallel$ *sy*

and $\mid ry \mid : \mid yt \mid \ = \ \mid ru \mid : \mid xy \mid$.

Show that $\mid xy \mid = \mid uy \mid$.

(iii)   Hence, or otherwise, prove that
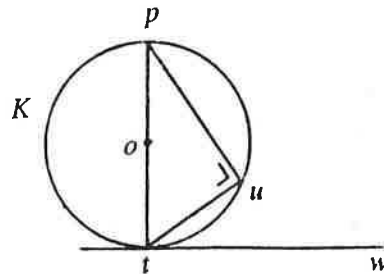*sy* bisects $\angle$ *xyt*.

4.   *(i)*   Prove that a line is a tangent to a circle at a point *t* on the circle if it is perpendicular to the diameter through *t*.

(ii)   *p*, *t* and *u* are points on a circle *K*, centre *o*.
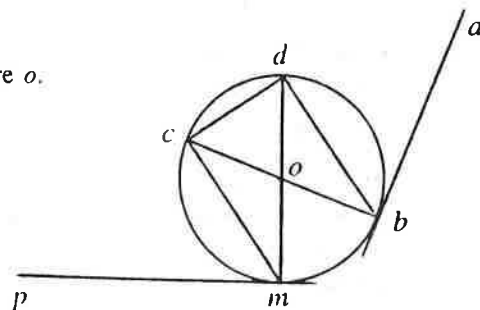
wt is the tangent at *t*.

[*pt*] is a diameter.

Prove that $\mid\angle$ *wtu* $\mid \ = \ \mid \angle$ *tpu* $\mid$.

(iii)   [*dm*] and [*cb*] are diameters of a circle, centre *o*.

ab and *pm* are tangents at *b* and *m* respectively.

Show that $\mid\angle$ *dba* $\mid = \mid\angle$ *pmc*$\mid$

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5. *a* (2, 5), *b* (−1, 1), and *c* (6, 2) are the vertices of a triangle.

Find | *bc* |.

Verify that *ab* ⊥ *ac*.

Find the equation of the line *K* through *a*, which is perpendicular to *bc* .

q is the mid-point of [*bc*].

Find the coordinates of *q*.

Verify that *q* is the circumcentre of Δ *abc*.

6.   *(a)*   Without using the Tables, construct an angle *A* such that

$$\cos A \ = \ \frac{2}{5}$$

Using the Tables, find the measure of the angle *B* such that

$$\cos B \ = \ 2\cos A, \quad 0 \le B \le 90\,°$$

   (b)   *t, x, u* and *y* are points on level ground, *x, u* and *y* in a straight line.

From *x* the direction of *t* is East 39° 46′ North.

From *y* the direction of *t* is West 68° 26′ North.

u is directly South of *t*.

| *xy* | = 95 m

Find | *tu* |, correct to the nearest metre.