INTERMEDIATE CERTIFICATE EXAMINATION, 1973

MATHEMATICS - LOWER COURSE - PAPER I

(150 marks)

MONDAY, 11 JUNE - MORNING, 9.30 to 12.00

SIX questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

1. A cylindrical vessel, internal diameter 30 cm and height 42 cm, is full of liquid. The liquid is being drawn from the vessel at the rate of $2\frac{3}{4}$ litres every hour.   How long will it take to draw all the liquid ?
    [1 litre = 1000 cm^3.   Take $\frac{22}{7}$ as an approximate value of π] .

2. The diagram shows a playing field, part of which has the shape of a rectangle, the ends being semi-circular.   The dimensions of the rectangle are 120 metres long and 98 metres wide.   Find:

    (a)   the length of the perimeter of the field,

   (b)   the area of the field in hectares, correct to one place of decimals.

   [1 hectare = 10^4 square metres.   Take $\frac{22}{7}$ as an approximate value of π] .
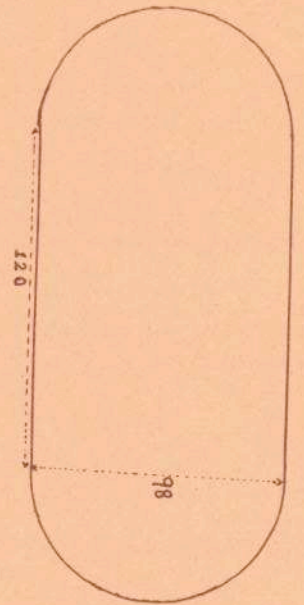
3. Prove that parallelograms on the same base and between the same parallels are equal in area.

    Construct a parallelogram abcd such that |ab| = 8 cm, |bc| = 5 cm, diagonal |ac| = 10 cm. Construct another parallelogram abef equal in area to abcd such that |ab| = 8 cm, |be| = 6·5 cm; explain your method.

4. (a) Three lines A, B, C are represented by a Venn diagram, as illustrated, and the shading indicates the empty set. Draw the lines A, B, C and mark the points p and q.
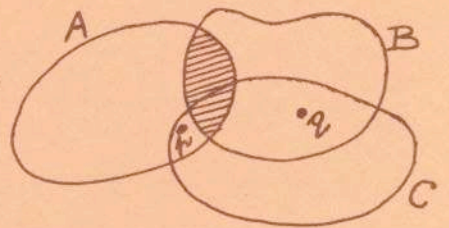
    (b) Represent by a Venn diagram the three lines P, Q, R and mark in your diagram the points x, y, z, t.
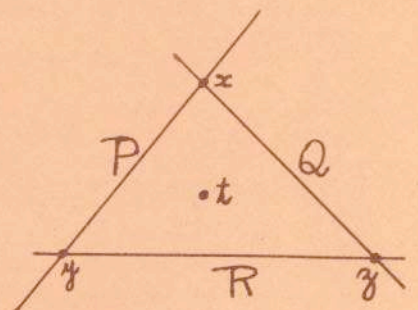
5. Each of the triangles X, Y, Z is the image of
the triangle T by either a translation, a central
symmetry or a reflection (axial symmetry).   Complete
each of the following statements by writing down a
*translation*, a *central symmetry* or a *reflection*, as
appropriate:
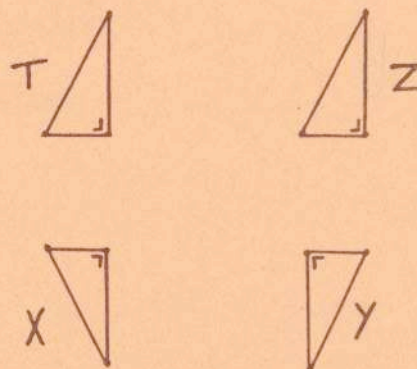
    (i) X is the image of T by ....

    (ii) Y is the image of T by ....

    (iii) Z is the image of T by ....

    Draw a diagram to show the construction of the
image in each case.

6. Draw a circle C of centre k and of radius 4 cm.   If p is a point such that $|kp|$ = 9 cm,
construct a tangent from p to C and give proof.   Calculate the length of the tangent to the
nearest centimetre and check your answer by measurement.

7. The line M is the perpendicular bisector (mediator) of $[cd]$, as
in diagram.   If gϵM, prove that $|gc|$ = $|gd|$.
    Hence, or otherwise, prove that the perpendicular bisectors of
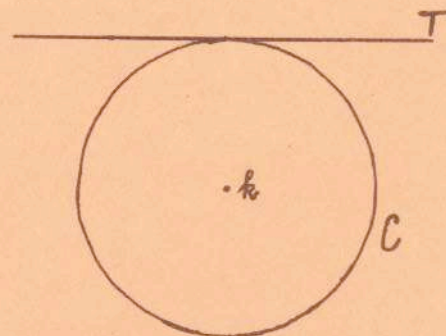the sides of a triangle are concurrent.
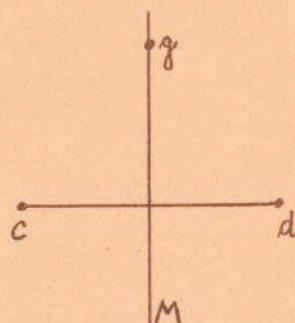
8. (a) Prove that the sum of any two opposite angles of a quadrilateral, inscribed in a
       circle, is two right angles.
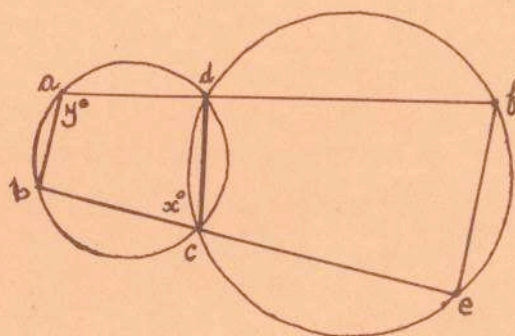
    (b) Two circles intersect at c and d, as in diagram.
       If the measure of $\angle dce$ = 110° , find the value
       of x and the value of y and prove that ab is
       parallel to fe.

9. T is a tangent to a circle C, centre k, as in diagram.
What is the image of C by

    (i) S_k, the central symmetry in k,

    (ii) S_T, the reflection (axial symmetry) in T ?

Illustrate your answer in each case by a diagram.
    Show also, by diagrams that $S_k \circ S_T \neq S_T \circ S_k$ with
respect to C.
    ($\neq$ means "not equal to")

10. (a) Use the tables to find 16 sin 40° and 2 cos 40° .

    (b) Liam wishes to get from a boat, b, offshore to a point c
       on shore, 400 metres away.   To do this he may swim from
       b to c or he may swim to a, the nearest point on shore
       and then run along the shore from a to c.   If Liam can
       swim at the rate of 25 metres per minute and run at
       eight times this rate, which route should he follow in
       order to reach c as quickly as possible ?   Explain
       your answer.   (See diagram)