INTERMEDIATE CERTIFICATE EXAMINATION, 1986

MATHEMATICS - HIGHER COURSE - PAPER II (300 marks)

MONDAY, 16 JUNE - MORNING, 9.30 to 12

SECTION A (100 marks)

Examination Number

Attempt all questions. You should not spend more than <u>50 minutes</u> on this section.
Answer each question by writing either (a), (b), (c), (d) in the box under each question number.
If you wish to change an answer, cross out your first choice and write your new answer near the
Mathematics tables may be obtained from the Superintendent.

THIS PAPER MUST BE ENCLOSED IN YOUR ANSWER BOOK

1. Which of the following is nearest to 1 ?

   [D]   (a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\frac{6}{7}$ (d) $\frac{8}{9}$

2. 42·5 is 85% of a number.   120% of the number is

   [C]   (a) 0·02   (b) 20   (c) 60   (d) 200

3. $\frac{0·4}{100}$ in scientific notation is

   [C]   (a) 0·004   (b) 4×10^2 (c) 4×10^{-3} (d) 4×10^{-2}

4. $\sqrt{1 - (0·6)^2}$ =

   [B]   (a) 0·36   (b) 0·8   (c) 0·64   (d) the square root of a negative number

5. Which of the following is true ?

   [A]   (a) $3 : 4 = 1·5 : 2$ (b) $3 : 4 = 2 : 3$
        (c) $3 : 4 = 4 : 5$ (d) $3 : 4 = 9 : 16$

6. A piechart represents the grades of 36 pupils.   Grade B is represented by an angle of 40°.   The number of B grades is:

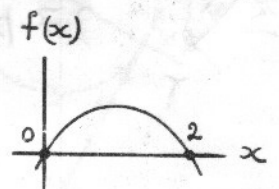   [A]   (a) 4   (b) 8   (c) 16   (d) 40

7. If $10^{1-x} = 50$, then 10^x is

   [A]   (a) $\frac{1}{5}$ (b) $\frac{1}{50}$ (c) 5   (d) 50

8. The factors of $x^2 - y^2 + (x + y)^2$ are

   [ ]   (a) $(x + y)(x - y)$ (b) $(x + y)(2y)$
   (c) $(x + y)(2x)$ (d) $(x + y)(2x - 2y)$.

9. The graph in the diagram represents the function
   $f : x \rightarrow$

   [D]   (a) $2 - x$ (b) $x(2 - x)$ (c) $-x(x + 2)$ (d) $x(x - 2)$

42.

10. $\log_{10} x$ has a characteristic of -2. Then x is a number between

(a) 9 and 1   (b) 0·9 and 0·1   (c) 0·09 and 0·01   (d) 0·009 and 0·001

11. The nth term of a sequence is $n + (n - 1)(n - 2)$. The first four terms are

(a) 1, 2, 3, 4 ....
(c) 2, 2, 1, 3 ....
(b) 0, 3, 5, 10 ....
(d) 1, 2, 5, 10 ...

12. A and B are not empty sets and $A \triangle B = A \cup B$. Then

(a) $A \setminus B = \{\ \}$ (b) $B \setminus A = \{\ \}$ (c) $A \cap B = \{\ \}$ (d) $A \subset B$.

13. The mean of 0, 1, 0, 1, 0, 1, 0, x, 1 is 1. Then x is

(a) 0   (b) 1   (c) 5   (d) 13

14. The coordinates $(x + 2, \ 4)$ and $(5, \ x + y)$ represent the same point. The value of y is

(a) 1   (b) 2   (c) 3   (d) 4.

15. If $f : x \to 10^x$ then which one of the following is false ?

(a) $f(3) = f(2) \times f(1)$ (b) $f(3) = f(6) \div f(3)$
(b) $f(3) = f(6) - f(3)$ (d) $f(3) = f(4) - 9f(3)$

16. The least value of $1 + (x - 1)^2$ is

(a) -1 (b) 0   (c) 1   (d) 2

17. Which one of the following is not a reflexive relation ?

(a) $\{(x, \ x)\}$ (b) $\{(y, \ y)\}$
(c) $\{(x, \ y), \ (y, \ x)\}$ (d) $\{(x, \ x) \ (y, \ y), \ (x, \ y), \ (y, \ x)\}$

18. A solid cube $3 \times 3 \times 3$ is cut in half as shown, (i.e. by a cut through the centre, parallel to a side). The surface area of each half is

(a) 18   (b) 27   (c) 36   (d) 54

19. $230_n \div 23_n$ is

(a) 10   (b) n (c) 10_n (d) n^2

20. If $\dfrac{1}{-x} > 3$, then x cannot be

(a) $-\frac{1}{3}$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{5}$ (d) $-\frac{1}{6}$
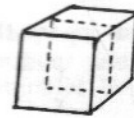
SECTION B (200 marks)

Attempt **QUESTION** 1 and **THREE** other questions

Marks may be lost if all your work is not clearly shown.

1. (a) If $\sqrt{\dfrac{1}{c}} = a^2 + b$, find the value of b, as accurately as the Tables allow, when $a = 0.5634$ and $c = 3.871$.

    (b) Dollars ($) were bought for IR£1000 on a day when the exchange rate was $1\$ = $ IR£1·10.
    The dollars were sold some time later and fetched IR£1045·45.
    What, to the nearest penny, was the exchange rate on the day the dollars were sold (i.e. $1\$ = $ IR£ ?) ?

2. (a) Factorise
    (i) $3x^2 + 13x - 30$.
    (ii) $6x^2 + 3xy - 2ax - ay$.
    (iii) $x^3 + 27y^3 + x + 3y$.

    (b) A large rectangle is divided into four smaller rectangles, the lengths of the sides being shown in the diagram.

    The area of the original rectangle is 35.

    Calculate a possible value for x, as accurately as the Tables allow.
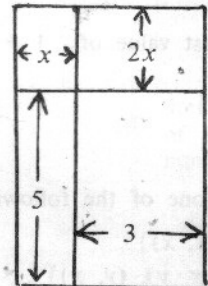
3. (a) f and g are the two functions
    $$f : x \to 3x - 4$$
    $$g : x \to 2x - 3.$$
    For what value of x is $f(x) = g(x)$?
    Express, in the form $x \to$,   the composite functions
    (i) $f \circ g$
    (ii) $g \circ f$
    and hence evaluate
    $$(f \circ g)(x) - (g \circ f)(x) - 2.$$

    (b) $S = \{1, 2, 3, 4\}$. T is $S \times S$.   Write down the elements of
    (i) $K : \{(x, y) \in T \mid x - y = 1\}$
    (ii) $H : \{(x, y) \in T \mid x - y = -1\}$
    (iii) $K \circ H$
    Plot the couples of $K, H, K \circ H$ on the plane.

4. The function $f : x \to 9 + x - x^2$, $x \in \mathbf{R}$, is defined in the domain $-2 \leqslant x \leqslant 4$.

(i) Complete the table

x	-3	-2	-1	0	1	2	3	4
$f(x)$	-3					7		-3

(ii) Draw the graph of the function f.

(iii) Find the values of x for which
$$f(x) = 0.$$

(iv) Use the graph to estimate the value of k for which $f(x) = k$ has one solution only.

(v) Use the same axes and scales to draw the graph of
$$g : x \to x + 1$$
and, hence, estimate $\sqrt{8}$.

5. A class had 30 pupils on the roll. During a five day week, Monday to Friday, a record was kept of the number of days each pupil was absent. The record is shown:

Number of days absent	0	1	2	3	4	5
Number of pupils	13	2	6	2	3	4

[i.e. 13 pupils missed no day, two missed one day, etc.]

(i) State the modal number of days missed.

(ii) Calculate the mean number of days missed per pupil, correct to one place of decimals.

(iii) How many attendances were recorded for the week ?

(iv) What was the greatest number of pupils that could have been present on any one of the days ?

(v) What was the least number of pupils that could have been present on any one of the days ?

6. (a) Solve for x
$$\frac{x + 3}{x + 2} + \frac{x - 3}{x - 2} = \frac{2x - 3}{x - 1}$$

(b) A person walks from p to q and back to p at a steady speed of 4 km/hour. A second person walks from p to q at a steady 3 km/hour and from q to p at a steady 5 km/hour. The second person took 11 minutes longer. Calculate the distance from p to q.

7. (a) If $x * y = (x + y) - 2(x - y)$, find $(x * y) * y$.

(b) Solve for x and y
$$3x - 2y = 19$$
$$\frac{x}{3} + \frac{y}{2} = 5.$$

(c) If $p = \log_{10} 2$ and $q = \log_{10} 3$, express, in terms of p and/or q.

(i) $\log_{10} 12$

(ii) $\log_{10} 5$

(iii) $\log_5 12$.