INTERMEDIATE CERTIFICATE EXAMINATION, 1982

MATHEMATICS - HIGHER COURSE - PAPER II (300 marks)

SECTION A (100 marks)

1. 10101_2 is not equal to

(a) 51_4 (b) 41_5 (c) 30_7 (d) 22_9

2. If $p:4 = 3:q$, then

(a) $3p = 4q$ (b) $\frac{1}{3}p = \frac{1}{4}q$ (c) $\frac{1}{4}p = \frac{1}{3}q$ (d) $pq = 12$

3. $\frac{1}{2} + \frac{1}{4} \times \frac{3}{4} =$

(a) $\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) $\frac{9}{16}$ (d) $\frac{11}{16}$

4. $(64)^{-\frac{2}{3}} =$

(a) 16 (b) - 16 (c) $\frac{1}{16}$ (d) $-\frac{1}{16}$

10.

5. The selling price of an article includes the cost price plus 20% VAT. In a sale this selling price is marked down 20%. The marked price is

(a) greater than the cost price
(b) equal to the cost price
(c) less than the cost price
(d) 40% of the cost price

6. 0·0146 expressed in the form $a \times 10^n$ where $1 \leqslant a < 10$ is

(a) $0{\cdot}146 \times 10^{-1}$
(b) $1{\cdot}46 \times 10^{-2}$
(c) $14{\cdot}6 \times 10^{-3}$
(d) 146×10^{-4}

7. A bus, 10 m in length and travelling due North at 10 m/s, begins to pass out an identical stationary bus also facing due North. The minimum, complete, passing-out time in seconds is

(a) 0·5
(b) 1
(c) 2
(d) 10

8. If $\#\ P = 6$ and $\#\ Q = 13$ where P and Q are sets, then $\#\ (P \cup Q)$ cannot be

(a) 10
(b) 13
(c) 16
(d) 19

9. If $\log_{10} 2 = x$, then $\log_2 10$ is

(a) $5x$
(b) $10x$
(c) $\frac{1}{x}$
(d) $\frac{1}{2x}$

10. The factors of $p^2 + q^2$ are

(a) $(a + b)(a + b)$
(b) $(a + b)(a - b)$
(c) $(a - b)(a - b)$
(d) none of these.

11. $(x^3 - y^3) \div (x - y) =$

(a) $x^2 + xy + y^2$
(b) $x^2 - xy + y^2$
(c) $x^2 + y^2$
(d) $x^2 - 2xy + y^2$

12. If $x * y = 2x - y^2$ then $(1 * 2) * 3$ equals

(a) -23
(b) -13
(c) -3
(d) -2

13. The roots of the equation $x^2 - x - 2 = 0$ are

(a) $\{1, 2\}$
(b) $\{-1, 2\}$
(c) $\{1, -2\}$
(d) $\{-1, -2\}$

14. To make a perfect square of $x^2 - 6x + 2$

(a) add 7
(b) subtract 7
(c) add $5x$
(d) add 4

15. The relation $R = \{(a, b), (a, c), (c, c), (b, a), (c, a)\}$ is

(a) reflexive
(b) symmetric
(c) transitive
(d) none of these

16. f is the function $x \to 3x + 2$. Then f^{-1}, i.e. the inverse function, is $x \to$

(a) $2 + x$
(b) $\frac{x - 2}{3}$
(c) $x - 2$
(d) $\frac{3}{x - 2}$

17. The nth term of a sequence is $\frac{n}{n - 2}$. The next term is

(a) $\frac{n + 1}{n - 1}$
(b) $\frac{n + 1}{n - 1}$
(c) $\frac{n}{n - 1}$
(d) $\frac{n}{n - 2} + 1$

18. The solution of the inequality $(x - 3)(x + 2) > 0$ is

(a) $-2 < x < 3$
(b) $-3 < x < 2$
(c) $x < -2$ and $x > 3$
(d) $x > 3$ and $x > -2$

19. If $V = \frac{1}{3}\pi r^2 h$, then r is equal to

(a) $\sqrt{\frac{3V}{\pi h}}$
(b) $\sqrt{\frac{V\pi h}{3}}$
(c) $\sqrt{3V\pi h}$
(d) $\sqrt{\frac{3Vh}{\pi}}$

20. If $f : x \longrightarrow 1 - x$ and $fg : x \longrightarrow x - 1$, where fg is the composite function, then $g : x \longrightarrow$

(a) $x - 2$ (b) $1 - 2x$ (c) $-x$ (d) $2 - x$

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MATHEMATICS - HIGHER COURSE - PAPER II

SECTION B (200 marks)

1. (a) Using Tables, P.20 - P.27, or otherwise, evaluate as accurately as the tables allow

$$p^2 - \sqrt{2q} - \frac{1}{r}, \text{ where}$$

$p = 4{\cdot}256$, $q = 0{\cdot}327$, $r = 0{\cdot}4528$.

(b) A book of raffle tickets sells for IR£1·20. The prizes in IR£ are 100, 85, 70, 55 and 40. If printing costs amount to IR£84, calculate the least number of books which must be sold
(i) to cover costs
(ii) to make a profit of IR£1000.

2. (a) Write down the factors of

$$6x^2 - 11x - 10$$

and, hence or otherwise, solve

(i) $6x^2 - 11x - 10 = 0$

(ii) $6(t - 1)^2 - 11(t - 1) - 10 = 0$.

(b) Solve $2x^2 - 3x - 7 = 0$ and give your answers correct to two places of decimals.

(c) Simplify $(p + q)^3 - (p - q)^3$ and show that $3p^2 + q^2$ is one of its factors.

3. The function f is defined on $\mathbf{R}$, such that, $f : x \longrightarrow 3x - 1$.
Find the value of (i) $f(2)$, (ii) $f(\frac{1}{2})$ and calculate $k \in \mathbf{N}$ such that $f(2) = kf(\frac{1}{2})$.
Investigate if

$$f(h) = kf(\tfrac{1}{h}).$$

g is another function defined on $\mathbf{R}$, such that $g : x \longrightarrow x^2 + 1$.
Investigate if

(iii) $g(x + 3) = g(x) + g(3)$

(iv) $fg(x) = gf(x)$, where these are composite functions.

4. Draw the graph of the function

$$f : x \longrightarrow 2 - 3x - x^2$$

in the domain $-5 \leqslant x \leqslant 2$, $x \in \mathbf{R}$.

Using the graph, estimate

(i) the values of x for which $f(x) = 0$
(ii) the range of values of x for which $f(x) \leq 0$
(iii) the values of x for which $5 - 3x - x^2 = 0$
(iv) the maximum value of $f(x)$
(v) the value of k for which $f(k + x) = f(k - x)$ for $x \in \mathbf{R}$.

5. (a) A train journey of 825 km was completed at a steady speed. Had the speed been reduced by 10 km/hour, the time for the journey would have been 45 minutes greater. Calculate the steady speed.

(b) If $A \Delta B = \{1, 5\}$, where $A = \{x, 2, 8, 9\}$, $B = \{2, 5, 8, y\}$, write out the value of (i) x (ii) y.
Hence write out the elements of the set C if

$$A \Delta B \Delta C = B.$$

6. (a) Solve $x^2 = 25 - y^2$
$x = 7 - y$.

(b) Justify $\log_t pq = \log_t p + \log_t q$.

If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, express in terms of a and b

(i) $\log_{10} 12$ (ii) $\log_{10} 6\frac{2}{3}$ (iii) $\log_{12} 6\frac{2}{3}$.

7. The fares paid by bus passengers in one day are shown in the following frequency distribution:

Fares in pence	10p	15p	20p	25p	30p	35p	40p
Number of passengers	20	30	20	40	10	10	20

(a) Construct a pie-chart showing the comparison between the number of fares

(i) of 15p or less

(ii) of 35p or more

(iii) other than (i) and (ii).

(b) Calculate the mean fare per passenger.

(c) By how much should the modal fare be raised to give a mean fare on the day of 23·6p ?