MATHEMATICS - HIGHER COURSE - PAPER I (300 marks)

THURSDAY 8 JUNE - MORNING 9.45 to 12.15

SECTION A (100 marks)

Examination Number

Attempt **all** questions.   You should not spend more than **50 minutes** on this section.

Answer each question by writing one of (a), (b), (c), (d) in the box under each question number.
If you wish to change an answer, cross out your first choice and write your new answer near the box.

Mathematics tables may be obtained from the Superintendent.

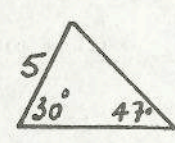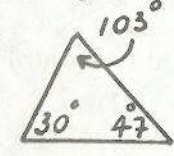### THIS PAPER **MUST** BE ENCLOSED IN YOUR ANSWER BOOK
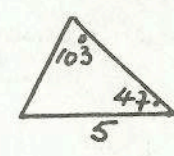
1.   A loss of 25% is made when an article is sold for £90.   The cost price is

    (a)   £112·50   (b)   £120   (c)   £115   (d)   £93·75

2. $0·01 \text{ m}^3$ expressed in cm^3 is

    (a)   1   (b)   10   (c)   100   (d)   10,000

3.   A certain perfume is sold in three sizes which one is the best value ?

    (a)   125 ml for 83p   (b)   75 ml for 49p   (c)   25 ml for 17p   (d)   all three are of equal value
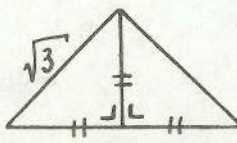
4.   Tap A fills a tank in half the time it takes tap B.   The two together fill it in 6 minutes. The number of minutes it takes tap A to fill the tank is

    (a)   9   (b)   12   (c)   18   (d)   36

5.   When an article is sold at the marked price, a shopkeeper makes a profit of 25%.   During a sale he gives a discount of 10% on the marked price.   The percentage reduction in profit is

    (a)   50   (b) $12\frac{1}{2}$ (c)   10   (d)   2·5

6. xyz is a triangle in which $|\angle xyz| = 30°$, $|\angle xzy| = 47°$ and $|yz| = 5$ cm.   Which one of the following is isometric (congruent) to the $\triangle xyz$?   (The diagrams are not drawn to scale.)

    (a)   (b)   (c)   (d)

7. $\vec{pq} = \vec{xy}$ where $p \neq x$.   Then

    (a) $(p,q) \in \vec{xy}$ (b) $p \in \vec{pq}$ (c) $(p,y) \uparrow (q,x)$ (d) $(p,q) = (x,y)$

8. (x,y) is a couple of the central symmetry, S_o, in the point o.   Which one of the following is not true ?

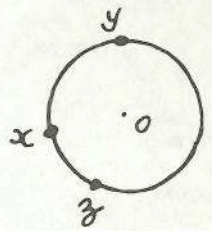   (a) $(x,o) \uparrow (o,y)$ (b) $(y,x) \in S_o$ (c) $x \in oy$ (d) $|xo| = 2|xy|$

9.   The area of the triangle is

    (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c)   2   (d) $\frac{9}{2}$

OVER →

10. p,q,r,s are the images of the four vertices of any given parallelogram under the projection parallel to K. Which one of the following is not always true ?
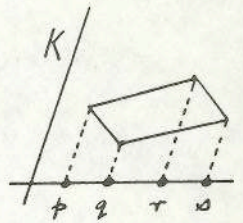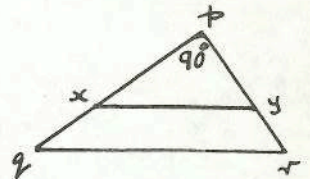
(a) $|pq| = |rs|$ (b) $|pq| = |qr|$

(c) $(p,r) \uparrow (q,s)$ (d) $(r,p) \uparrow (s,q)$

11. o is the centre of the circle and x,y,z are any three points of the circle. Which one of the following is false ?
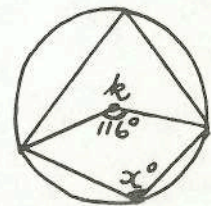
(a) The perpendiculars to the tangents at x,y,z are concurrent at o.
(b) The perpendicular bisectors of $[xy]$, $[xz]$, $[yz]$ are concurrent at o.
(c) $o \in yz \Rightarrow |\angle yxz| = 90°$.
(d) The bisector of $\angle yxz$ must contain o.

12. $|px| = 4$, $|xq| = 2$, $|py| = 3$ and $xy \parallel qr$. Then $|qr|$ is
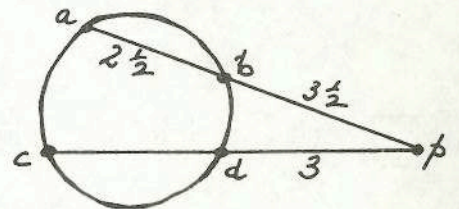
(a) $7\frac{1}{2}$ (b) $4\frac{4}{5}$ (c) $3\frac{1}{3}$ (d) $2\frac{1}{2}$

13. k is the centre of the circle. The value of x is

(a) 116   (b) 64   (c) 132   (d) 122

14. $|pb| = 3\frac{1}{2}$, $|ba| = 2\frac{1}{2}$, $|pd| = 3$. Then $|cd|$ is

(a) 2   (b) 4   (c) $\dfrac{2\frac{1}{2} \times 3\frac{1}{3}}{3}$ (d) 3

15. The slope of the line perpendicular to $2x - 5y + 3 = 0$ is

(a) $-\frac{5}{2}$ (b) $-\frac{2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{2x}{5y}$

16. The lines $x - y + 3 = 0$ and $2x - y + 5 = 0$ intersect at the point

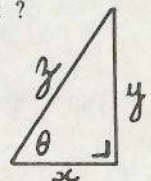(a) $(-2, 1)$ (b) $(-2, -1)$ (c) $(-2\frac{2}{3}, \frac{1}{3})$ (d) $(2, -1)$

17. Which one of the following does not map $\{(-3, 4), (2, 2)\}$ onto $\{(-2, -2), (3, -4)\}$?

(a) a translation   (b) a central symmetry   (c) a rotation   (d) an axial symmetry

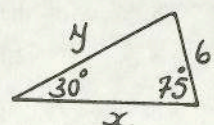18. If $\sin^2 x = 1$ and if $0° \leqslant x \leqslant 360°$, then the set of values of x is

(a) $\{90°\}$ (b) $\{270°\}$ (c) $\{90°, 270°\}$ (d) $\{0°, 180°, 360°\}$

19. The right angle is indicated in the diagram. Which one of the following is correct ?

(a) $y = x \tan \theta$ (b) $z = x \cos \theta$

(c) $x = y \tan \theta$ (d) $z = y \sin \theta$

20. Which one of the following is true ?

(a) $\dfrac{y}{\sin 75°} = 3$ (b) $\dfrac{x}{\sin 75°} = 12$

(c) $\dfrac{y}{\sin 75°} = \frac{1}{3}$ (d) $\dfrac{x}{\sin 75°} = \frac{1}{12}$

SECTION B (200 marks)
Attempt **QUESTION** 1 and **THREE** other questions

1. (a) Central heating uses 1260 litres of oil to heat a house for 35 days.   If oil costs 8·64p a litre
      and the heating is switched on for 9 hours every day, find the cost per hour, to the nearest
      penny, of heating the house for the 35 days.

   (b) When the rate of exchange is £1 = 4·35 German marks, it costs £4515 to purchase marks in
      order to import a car from Germany.

      When the rate of exchange drops to £1 = 4·30 marks, find the increased cost, to the nearest £,
      of importing the same car.

                                                                                (50 marks)

2. *abc* and *dce* are two triangles in which

   $$|ba| = |cd|$$
   $$|bc| = |ce|$$
   $$|\angle abc| = |\angle dce| .$$
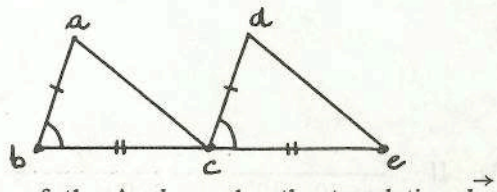
   If *b,c,e* are collinear, prove that the $\triangle$ *dce* is the image of the $\triangle$ *abc* under the translation $\vec{bc}$.

   xyz is a triangle such that $|xy| = |xz|$.   If *p* and *q* are the midpoints of [*xy*] and [*xz*], respectively,
   prove that $|pz| = |qy|$.

                                                                                (40 marks)

3. Prove that a circle is its own image under the axial symmetry in any line through its centre.

   D is a line through the centre of the circle which is perpendicular to
   the chord [*ab*].   Prove that *D* bisects [*ab*].
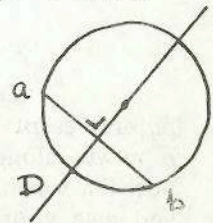
   A radius of a circle is 5 cm in length.   Find the length of a chord which
   is 3 cm from the centre of the circle.

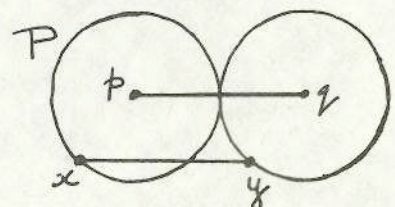   (40 marks)

4. *A, B, K* are lines such that *A*||*B* and *A*⊥*K*.
   In each of the following give one transformation
   which equals
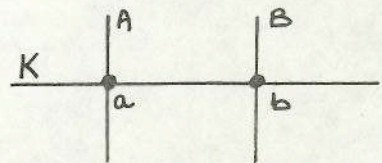
   (i) $S_b \ o \ S_a$, where S_b, S_a are central symmetries in *b* and *a*,
       respectively

   (ii) $S_B \ o \ S_K$, where S_B, S_K are axial symmetries in *B* and *K*,
       respectively

   and prove that $S_B \ o \ S_A$ is a translation.

   P and *Q* are two circles having radii of equal length and having
   centres at *p* and *q*, respectively.   *x* is any point of *P* and a
   line through *x*, parallel to *pq*, meets *Q* at *y* as in diagram.

   Prove that $|xy| = |pq|$.

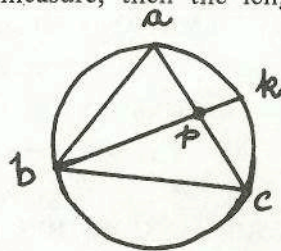   (50 marks)
                                                                                OVER →

5. Prove that if the angles of two triangles are equal in measure, then the lengths of their corresponding sides are proportional.

In the diagram bk bisects the $\angle abc$.   Prove that

$$|ba| \cdot |bc| = |bk| \cdot |bp|$$

(50 marks)

6. The points a, b, c have coordinates $(-2, 1)$, $(1, -3)$, $(3\frac{1}{2}, 2)$, respectively.
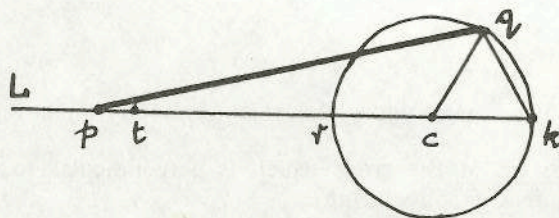
(i) Calculate the area of the $\triangle abc$, the length $|ab|$ and hence find the distance from c to the line ab.

(ii) Write down the slope of ab and then find the equation of the line K, through c, which is perpendicular to ab.   Find the coordinates of the point where K intersects ab.

(50 marks)

7. (i) If $\dfrac{3}{\sin 30° \, 40'} = \dfrac{y}{\sin 42° \, 51'}$,

calculate y as accurately as the tables allow.

(ii)

$|pq|$ represents an iron bar.   As q moves around the circle, of centre c and of radius 5 cm, p moves along the line L so that when q is at k, p is at t.   The diagram shows the position when the $\triangle qck$ is equilateral and $|\angle pqr| = 17° \, 30'$.   If $|tr| = |rk|$, calculate $|pt|$ and give your answer correct to the nearest whole number.

(Note: If $A + B = 180°$, then $\sin A = \sin B$).

(50 marks)