

## INTERMEDIATE CERTIFICATE EXAMINATION, 1975

## MATHEMATICS - HIGHER COURSE - PAPER II

MONDAY, 16 JUNE - MORNING, 9.30 to 12

SIX questions to be answered.  
All questions are of equal value.  
Mathematics Tables may be had from the Superintendent.

1. (a) Calculate, to the nearest penny, the compound interest on £25 for three years at 11% per annum.
- (b) A Hot-tap can fill a bath in 2 minutes. The Cold-tap can fill the bath in 1 minute. If both taps are filling the bath at the same time, find
- how much of the bath would have water in it after  $\frac{1}{2}$  minute,
  - how long it would take to fill the bath.
2. (a) Let  $U = \{x \mid 0 < x \leq 9, x \in N\}$ . Write out the elements of each of the following where  $P, E, K$  are subsets of  $U$ :
- $P = \{x \mid x \text{ is prime}\}$
  - $E = \{x \mid x \text{ is even}\}$
  - $K = \{x \mid x \text{ is odd}\}$
  - $P \cap E$
  - $P \cap K$
- (b) If  $A, B, C, D$  and  $Q$  are sets, is each of the following statements true? Give an example in each case to illustrate your answer.
- $Q \cap A = Q \cap B \Rightarrow A = B$ ,
  - $Q \times C = Q \times D \Rightarrow C = D$ .
- (c) If  $x$  is a number between 0 and 1 (i.e.  $0 < x < 1$ ), write down the following in order, beginning with the least:
- $$x, \quad \sqrt{x}, \quad \frac{1}{x}, \quad \frac{1}{\sqrt{x}}.$$

3. (a) Let  $A = \{x \mid -3 \leq x \leq 3, x \in Z\}$ . Find the values of  $x \in A$  which satisfy each of the following:
- $3 - 2x \geq 5$
  - $\frac{1}{x} > \frac{1}{2}$
  - $2x^2 - 1 < 5$ .
- (b) Factorise  $x^2 - 4x - 21$ .  
Hence, or otherwise, find the solution set of  $x^2 - 4x - 16 \leq 5$  and graph your solution on the number line.

4. (a) Solve the simultaneous equations

$$3x - y = 6$$

$$x = 3y - 2.$$

What is the geometrical meaning of your answer?

- (b) The operation  $*$  is defined by

$$a * b = 2a + b, \text{ where } a, b \in R.$$

Answer each of the following giving a reason in each case:

- Is  $N$ , the set of natural numbers, closed under  $*$ ?
- Is the operation  $*$  commutative?
- Is the operation  $*$  associative?
- Does  $a * b = 0 \Rightarrow$  either  $a = 0$  or  $b = 0$ ?

5. (a) Find the common difference of the arithmetic series

$$\frac{3}{4} + 1\frac{1}{4} + 1\frac{3}{4} + \dots$$

and find the sum of  $n$  terms.  
How many terms must be summed to make 42 ?

- (b) The  $n$ th term of a sequence is

$$n + (n - 1)(n - 2)(n - 3).$$

Write down the first four terms and verify that the sequence is not geometric.

6. Draw the graph of the function

$$f : x \rightarrow x^2 + 2x - 8 (= y)$$

in the domain  $-5 \leq x \leq 3$ .  
Use your graph to find:

- the domain of  $x$  for which  $f(x)$  is positive and increasing,
- the values of  $x$  for which  $x(x + 2) = 6$ ,
- the domain of  $x$  for which  $-2 \leq f(x) \leq 5$ .

7. A gardener had 1440 onions to plant in rows having  $x$  onions per row. He found, however, that  $x$  onions would not fit in a row so he put  $(x - 5)$  onions in each row and he then needed 24 extra rows. Find the value of  $x$  and then calculate the number of rows he sowed.

8. (a) Simplify each of the following:

(i)  $16^{\frac{1}{2}}$

(ii)  $(-\frac{1}{4})^{-2}$

(iii)  $\log_2 \sqrt{2}$ .

- (b) If  $\log_{10} 3 = a$  and  $\log_{10} 5 = b$ , express each of the following in terms of  $a$  and  $b$ :

(i)  $\log_{10}(\frac{5}{3})$

(ii)  $\log_{10} \sqrt{45}$

(iii)  $\log_3 5$ .

- (c) Solve the equation

$$1 + \log_2(3x + 1) = \log_2(2x + 1).$$

9. (a) "The average (mean) of a set of numbers is always an element of the set".  
Is the above statement true? Give a reason.  
The average of the three numbers  $x, y, z$  is 1 and the average of the four numbers  $x, y, z, t$  is also 1. Calculate the value of  $t$ .

- (b) The following distribution table gives the number of potatoes per plant and the corresponding number of plants:

Number of Potatoes per plant	1	2	3	4	5	6	7	8	9	10
Number of plants	1	1	2	3	10	20	20	14	5	4

Calculate the average (mean) number of potatoes per plant and illustrate the data by a frequency polygon.

10. Using the same axes and the same scales graph the inequalities

$$x + y \geq 1; \quad x - y - 1 \geq 0; \quad 0 \leq x \leq 2.$$

- Indicate on your graph the set  $P$  such that the coordinates of every point of  $P$  simultaneously satisfy the above inequalities.
- For  $(x, y) \in P$ , find the maximum value of  $\frac{1}{2}x + y$ .
- For  $(x, y) \in P$ , indicate by the letter  $Q$  the set  $\{(x, y) \mid 4(x^2 + y^2) \geq 9\}$ .