

INTERMEDIATE CERTIFICATE EXAMINATION, 1975

MATHEMATICS - HIGHER COURSE - PAPER I

WEDNESDAY, 11 JUNE - MORNING, 9.45 to 12.15

SIX questions to be answered.

All questions are of equal value.

Mathematics Tables may be had from the Superintendent.

1. (a) If $g = 9.8$ and $r = 4000$, calculate v , correct to two significant figures, where

$$\frac{v^2}{r} = \frac{g}{291}.$$

- (b) x , y and z are three numbers such that $x : y = 2 : 3$ and $x : z = 5 : 7$. What is the ratio between y and z ?

2. Water is flowing through a cylindrical pipe, of diameter 40 cm, at the rate of 10 kilometres per hour, into a rectangular reservoir of length 100 metres and breadth 80 metres. By how much, to the nearest centimetre, will the height of the water rise in the reservoir in 15 minutes?

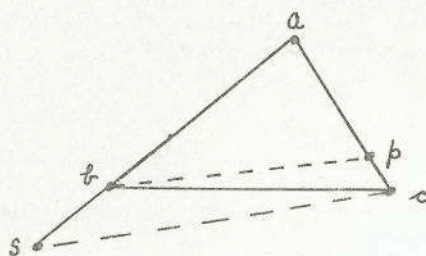
[Take 3.14 as an approximation for π].

- If the pipe had been 80 cm in diameter and the water flowing at the same rate, what would be the rise?

3. Prove that two triangles on the same base and between the same parallels are equal in area.

abc is a triangle and p is a point on the side $[ac]$.
 cs is drawn parallel to pb intersecting ab at s .

Show that the triangles asp and abc are equal in area.

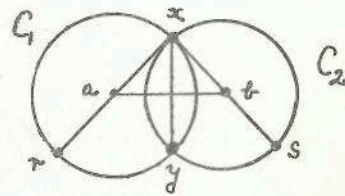


4. a and b are the centres of two circles C_1 and C_2 , respectively. What are the images of C_1 and C_2 by the reflection (axial symmetry) in ab ?

If the two circles intersect at x and y , prove that $ab \perp xy$ and that ab bisects the line segment $[xy]$.

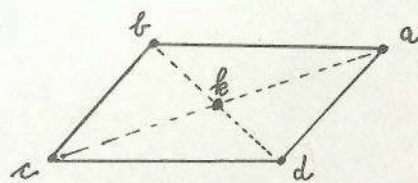
$[xr]$ and $[xs]$ are diameters of the circles, as in diagram. Prove that the points r , y , s are collinear and that $|rs| = 2|ab|$.

Prove



5. Prove that the diagonals of a parallelogram bisect each other. The diagonals of a parallelogram $abcd$ meet at k . Construct the image of the triangle akd by the translation \vec{cd} after S_k , the central symmetry in k .

The point t is the fourth vertex of the parallelogram $dkct$. What composition of a central symmetry and a translation gives the triangle cdt as the image of the triangle cdk ?



6. $[ab]$ and $[cd]$ are two chords of a circle. When produced the two chords meet in a point k , outside the circle. Prove that

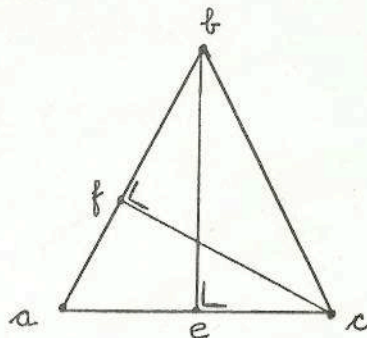
$$|ka| \cdot |kb| = |kc| \cdot |kd|.$$

bac is an isosceles triangle having $|ba| = |bc| = 9$ cm and $|ac| = 6$ cm, as in diagram.

If $cf \perp ba$ and $be \perp ac$, prove that

- (i) the quadrilateral $bfec$ is cyclic,
- (ii) e is the midpoint of $[ac]$.

Calculate $|af|$.



7. The points $a(1, 1)$, $b(4, 2)$ and $c(2, 4)$ are the vertices of a triangle. Show that this triangle is isosceles.

A translation \vec{t} maps $(-1, 1) \rightarrow (0, 0)$ and S_X is the reflection (axial symmetry) in the line $y = 0$. Find the coordinates of the vertices of the image of the triangle abc under the composition S_X after \vec{t} .

Verify that this image is also an isosceles triangle.

8. (a) The equation $x^2 + y^2 = 25$ represents a circle. Write down the length of its radius and the coordinates of its centre.

(b) $x - 2y + 5 = 0$ is the equation of a line L . Find the equation of the line M which is perpendicular to L and passes through the origin. Find the coordinates of the point where L meets M . What is the equation of the circle, centre $(0, 0)$, which has L as a tangent?

9. Using the same axes and the same scales, sketch the graphs of the functions:

$$f : x \rightarrow \sin x \quad \text{and} \quad g : x \rightarrow 2\sin x \quad \text{in the domain } 0 \leq x \leq 2\pi.$$

From the graphs write down

- (i) the range of f ,
- (ii) the range of g ,
- (iii) the domain of x for which $f(x) \geq g(x)$.

10. (a) If $\sin x = 0.9102$, $x < 90^\circ$, find $\cos x$.

(b) Two ships X and Y set sail from the same port at the same time. X sailed in a direction 37° South of West and Y in a direction 22° South of East. After a certain time X and Y were 2 kilometres apart and X was due West of Y . How far had the ship X sailed? Write your answer correct to the nearest km.