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INTERMEDIATE CERTIFICATE EXAMINATION, 1974

MATHEMATICS – HIGHER COURSE – PAPER II

(300 marks)

TUESDAY, 18 JUNE – MORNING, 9.30 to 12

Six questions to be attempted.
All questions are of equal value.
Mathematics tables may be obtained from the Superintendent.

1. (a) Calculate the compound interest on £500 for 3 years at 12% per annum. Give your answer correct to the nearest penny.

   (b) A dealer makes a profit of 25% by selling used cars at £400 each. Calculate his total profit on the sale of 40 cars.
   When he reduces his selling price by 4% per car, the number he sells increases by 50%. Calculate the percentage increase in his total profits.

2. (a) Use Venn diagrams to illustrate
   (i) $A \cap B'$ (ii) $A \setminus (A \setminus B)$ (iii) $(A \cup B) \setminus (A \cap B)$.

   (b) A party of 175 tourists visited Connaught. 5 visited Galway and Sligo but not Mayo. 15 visited Galway and Mayo but not Sligo. 30 visited Galway only. 15 visited Mayo only and 50 visited Sligo only. 20 did not visit any of the three counties Galway, Mayo, Sligo. If Galway was visited by 60 tourists, how many tourists visited (i) all three counties (ii) two only of the three counties (iii) one only of the three counties.

3. (a) x and y are any two numbers such that $1 \leqslant x \leqslant 2$ and $3 \leqslant y \leqslant 4$. Say which of the following statements are true and which are false:
   (i) $x - y > 0$ (ii) $1 - x < 1 - y$ (iii) $\frac{1}{y} < \frac{1}{x}$
   (iv) $x^2 + y^2 \leqslant 20$ (v) $x^2 - y^2 \geqslant -4$.

   (b) Find the solution set of the inequality $(3x - 2)(x + 2) > 3$ and show the solution set on the number line.

4. (a) $P = \{k, \sin 35°, 9\}$, $Q = \{9, k\}$. If X is a set such that $P \cap X = Q$, write down the elements of a possible set X. Is your answer the only possible set ? Give a reason.

   (b) Let $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 5\}$. Write down the elements of $A \times B$ and $A \times C$. Verify the $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

   (c) Let S be the set of all subsets of $\{x, y\}$. Graph the relation "is a subset of" on the set S. Say, giving a reason, whether or not this relation is (i) reflexive, (ii) symmetric, (iii) transitive.

5. (a) The nth term of a sequence is $an^2 + bn + 5$. If the first term is 11 and the second term is 27, find the value of a and the value of b. Find the third term. Verify that the sequence is neither arithmetical nor geometrical.

   (b) Find the greatest value of r, where $r \in \mathrm{N}$, for which $(1{\cdot}4)^r < 10$.

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6. Graph the function $f : x \to 4 - 3x - x^2 (=y)$ for values of x in the domain $-5 \leqslant x \leqslant 2$. Using your graph or otherwise answer the following:

(i) Why is the line $x = -1\frac{1}{2}$ an axis of symmetry of the graph ?

(ii) What is the maximum value of $f(x)$ i.e. y ?
What value of x gives this maximum ?

(iii) For what domain is $f(x)$ positive and decreasing ?

(iv) Find the solution set of $4 - 3x - x^2 = 2$.

(v) Why is there no real solution to the equation $x^2 + 3x + 4 = 0$?

7. A rectangular photograph was placed on a piece of cardboard so as to leave a $2\frac{1}{2}$ cm border of cardboard around the photograph. One side of the photograph was 1 cm longer than the other. The area of the cardboard was 156 cm^2. Find the length and breadth of the photograph.

8. (a) The length of a rod correct to *three* significant figures is 34·7 cm. If the length of the rod were measured correct to *four* significant figures, find (i) the maximum, (ii) the minimum measurement it could be.

(b) Write down all the binary whole numbers between 1101 and 10011.

(c) Find the value of x in each of the following:
(i) $\log_2 x = -3$, (ii) $\log_x 27 = 3$, (iii) $\log_{10} x = 1 - m$ and $\log_{10} 4x = 1 + m$.

9. Airport records showed the following numbers of aircraft which took off each day over a fifty day period.

5	2	3	6	8	2	10	9	8	3
7	8	8	4	1	6	4	5	7	8
0	6	10	5	9	2	7	1	10	2
3	2	5	0	1	8	8	8	7	9
4	4	6	3	2	1	9	2	3	9

(a) Construct the frequency table and calculate the mean number of aircraft taking off daily.

(b) Construct the cumulative frequency table and draw a cumulative frequency curve. From the curve estimate the median number of aircraft taking off daily.

10. Using the same axes and the same scales graph the following inequalities:

(i) $x + y \leqslant 6$,

(ii) $x + 2y \leqslant 7$,

(iii) $x \geqslant 0$,

(iv) $y \geqslant 0$.

Shade the set of points S which simultaneously satisfy all four inequalities.
Why does (1, 3) belong to the set S ?
Find the point (x, y) in S which makes $2x + 3y$ a maximum and calculate this maximum value.